each series being set and regulated* by observers in the corresponding system. The time at which the event in question takes place may be denoted by t if determined by the clocks belonging to system S and by t' if determined by the clocks of system S'.

For convenience the two systems S and S' are chosen so that the axes OX and $O'X'$ lie in the same line, and for further simplification we choose, as our starting-point for time measurements, t and t' both equal to zero when the two origins come into coincidence.

The specific problem now before us is as follows: If a given kinematical occurrence has been observed and described in terms of the variables x', y', z' and t', what substitutions must we make for the values of these variables in order to obtain a correct description of the *same* kinematical event in terms of the variables x, y, z and t? In other words, we want to obtain a set of transformation equations from the variables of system S' to those of system S. The equations which we shall present were first obtained by Lorentz, and the process of changing from one set of variables to the other has generally been called the Lorentz transformation. The significance of these equations from the point of view of the theory of relativity was first appreciated by Einstein.

Deduction of the Fundamental Transformation Equations.

34. It is evident that these transformation equations are going to depend on the relative velocity V of the two systems, so that we may write for them the expressions

$$x' = F_1(V, x, y, z, t),$$

*We may think of the clocks as being set in any of the ways that are usual in practice. Perhaps the simplest is to consider the clocks as mechanisms which have been found to "keep time" when they are all together where they can be examined by one individual observer. The assumption can then be made, in accordance with our ideas of the homogeneity of space, that they will continue to "keep time" after they have been distributed throughout the system.

$$y' = F_2(V, x, y, z, t),$$
$$z' = F_3(V, x, y, z, t),$$
$$t' = F_4(V, x, y, z, t),$$

where F_1, F_2, etc., are the unknown functions whose form we wish to determine.

It is possible at the outset, however, greatly to simplify these relations. If we accept the idea of the homogeneity of space it is evident that any other line parallel to OXX' might just as well have been chosen as our line of X-axes, and hence our two equations for x' and t' must be independent of y and z. Moreover, as to the equations for y' and z' it is at once evident that the only possible solutions are $y' = y$ and $z' = z$. This is obvious because a meter stick held in the system S' perpendicular to the line of relative motion, OX', of the system can

not possible for v>c as y'=iy z'=iz

be directly compared with meter sticks held similarly in system S, and in accordance with the first postulate of relativity they must agree in length in order that the systems may be entirely symmetrical. We may now rewrite our transformation equations in the simplified form

I dont agree: a measurement is not a physical law

$$x' = F_1(V, t, x),$$
$$y' = y,$$

only valid for v<c

$$z' = z,$$
$$t' = F_2(V, t, x),$$

and have only two functions, F_1 and F_2, whose form has to be determined.

To complete the solution of the problem we may make use of three further conditions which must govern the transformation equations.

35. Three Conditions to be Fulfilled. In the first place, when the velocity V between the systems is small, it is evident that the transformation equations must reduce to the form that they had in Newtonian mechanics, since we know both from measurements and from everyday experience that the Newtonian concepts of space and time are correct as long as we deal with slow velocities. Hence the limiting form of the equations as V approaches zero will be (cf. Chapter I, equations (3), (4), (5), (6))

$$
\begin{aligned}
x' &= x - Vt, \\
y' &= y, \\
z' &= z, \\
t' &= t.
\end{aligned}
$$

first postulate can not exclude some change due to passing the strong singularity v=c

36. A second condition is imposed upon the form of the functions F_1 and F_2 by the first postulate of relativity, which requires that the two systems S and S' shall be entirely symmetrical. Hence the transformation equations for changing from the variables of system S to those of system S' must be of exactly the same form as those used in the reverse transformation, containing, however, $-V$ wherever $+V$ occurs in the latter equations. Expressing this requirement in mathematical form, we may write as true equations

$$
\begin{aligned}
x &= F_1(-V, t', x'), \\
t &= F_2(-V, t', x'),
\end{aligned}
$$

only valid for v<c

or have to write F1 and F2 so works for v>c and change sign

where F_1 and F_2 must have the same form as above.

37. A final condition is imposed upon the form of F_1 and F_2 by the second postulate of relativity, which states that the velocity of a beam of light appears the same to all observers regardless of the motion of the source of light or of the observer. Hence our transformation equations must be of such a form that a given beam of light has the same velocity, c, when measured in the variables of either system. Let us suppose, for example, that at the instant $t = t' = 0$, when the two origins come into coincidence, a light impulse is started from the common point occupied by O and O'. Then, measured in the coördinates of either system, the optical disturbance which is generated must spread out from the origin in a spherical form with the velocity c. Hence, using the variables of system S, the coördinates of any point on the surface of the disturbance will be given by the expression

$$x^2 + y^2 + z^2 = c^2t^2, \tag{7}$$

while using the variables of system S' we should have the similar expression

$$x'^2 + y'^2 + z'^2 = c^2t'^2. \tag{8}$$

Thus we have a particular kinematical occurrence, the spreading out of a light disturbance, whose description is known in the variables of either system, and our transformation equations must be of such a form that their substitution will change equation (8) to equation (7). In other words, the expression $x^2 + y^2 + z^2 - c^2t^2$ is to be an invariant for the Lorentz transformation. enough they are proportional and pseudo invariant

38. The Transformation Equations. The three sets of conditions which, as we have seen in the last three paragraphs, are imposed upon the form of F_1 and F_2 are sufficient to determine the solution of the problem. The natural method of solution is obviously that of trial, and we may suggest the solution:

$$x' = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}(x - Vt) = \kappa(x - Vt), \tag{9}$$

$$y' = y, \tag{10}$$

$$z' = z, \tag{11}$$

$$t' = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\left(t - \frac{V}{c^2}x\right) = \kappa\left(t - \frac{V}{c^2}x\right), \tag{12}$$

where we have placed κ to represent the important and continually recurring quantity $\dfrac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$.

It will be found as a matter of fact by examination that these solutions do fit all three requirements which we have stated. Thus, when V becomes small compared with the velocity of light, c, the equations do reduce to those of Galileo and Newton. Secondly, if the equations are solved for the unprimed quantities in terms of the primed, the resulting expressions have an unchanged form except for the introduction of $-V$ in place of $+V$, thus fulfilling the requirements of symmetry imposed by the first postulate of relativity. And finally, if we substitute the expressions for x', y', z' and t' in the polynomial $x'^2 + y'^2 + z'^2 = c^2t'^2$, we shall obtain the expression $x^2 + y^2 + z^2 - c^2t^2$ and have thus secured the invariance of $x^2 + y^2 + z^2 - c^2t^2$ which is required by the second postulate of relativity.

We may further point out that the whole series of possible Lorentz transformations form a group such that the result of two successive transformations could itself be represented by a single transformation provided we picked out suitable magnitudes and directions for the velocities between the various systems.

It is also to be noted that the transformation becomes imaginary for cases where $V > c$, and we shall find that this agrees with ideas obtained in other ways as to the speed of light being an upper limit for the magnitude of all velocities.

v>c has been implicitly excluded by wrong arguments

Further Transformation Equations.

39. Before making any applications of our equations we shall find it desirable to obtain by simple substitutions and differentiations a series of further transformation equations which will be of great value in our future work.

By the simple differentiation of equation (12) we can obtain

$$\frac{dt'}{dt} = \kappa\left(1 - \frac{\dot{x}V}{c^2}\right), \tag{13}$$