Ludwik Silberstein 1914

Comments to Silberstein *The theory of relativity* 1914
(can be downloaded at donation at https://archive.org/details/theoryofrelativi00silbrich)
Silberstein uses at start , as Einstein , light signals going to a point A and return in the direction of movement between two systems , an in perpendicular direction, which , as in the case of Einstein seems from start exclude v>c, as in the first case the light will not reach the point A and in perpendicular direction $\sqrt{c^2 - v^2}$ is imaginary. But even if that procedure seemingly physically exclude v>c, Einstein could have found that (Pilotti 2020, pp. 127-130) so also Silberstein, which is easily seen below.

Up to here Silberstein makes a detailed and physical argument, albeit the process presumes v<c.

But there is no reason as on p. 110 to here choose just

$$\alpha = 1$$

which gives the ordinary LT

(9) gives

$$ds^2 = \alpha^2 ds'^2$$

And e.g. Rindler's derivation gives

$$ds^2 = \pm ds'^2$$

where + of course gives ordinary LT
But – gives GLT
For v>c
It is easily seen here if we take,

But, by the first of formulae (4) and by the above convention as to the origin of time-reckoning at O',

$$t(0,\ t') = \alpha\gamma t'.$$

Hence

$$t \equiv t(x',\ t') = \alpha\gamma\left(t' + \frac{v}{c^2}x'\right), \tag{7}$$

which is the required connexion. Substituting here x' from (5) and remembering that $\beta^2 + 1/\gamma^2 = 1$, we shall obtain t' in terms of t, x. " also $1+\gamma^2\beta^2=\gamma^2$ and that $\frac{v^2}{c^2}=\beta^2$, or $1+\gamma^2\frac{v^2}{c^2}=\gamma^2$

Thus, the complete set of formulae connecting the S'- with the S-time and coordinates will be

$$\left.\begin{aligned} x' = \frac{\gamma}{\alpha}(x - vt)\,; \quad y' = \frac{1}{\alpha}y\,; \quad z' = \frac{1}{\alpha}z \\ t' = \frac{\gamma}{\alpha}\left(t - \frac{v}{c^2}x\right). \end{aligned}\right\} \tag{8}$$

Conversely, resolving these equations with respect to t, x, y, z, or simply copying (7) and using it to eliminate t from the first equation,

$$\left.\begin{aligned} x = \alpha\gamma(x' + vt')\,; \quad y = \alpha y'\,; \quad z = \alpha z' \\ t = \alpha\gamma\left(t' + \frac{v}{c^2}x'\right). \end{aligned}\right\} \tag{9}$$

Notice that, disregarding α, the set (9) follows from (8), and *vice versa*, by simply interchanging x, y, z, t with x', y', z', t' and by writing $-v$ instead of v. Now $\mathbf{v}$ being the velocity of S' relative to S, $-\mathbf{v}$ will be the velocity of S relative to S'.* As to c, it is common to both systems, and $\gamma(v) = \gamma(-v) = (1 - v^2/c^2)^{-\frac{1}{2}}$. Thus, there is **reciprocity** between the two systems of reference, except for the common arbitrary coefficient which is α^{-1} in the

* In fact, what we call the velocity of S relative to S' is the vector whose components are the derivatives of x', y', z' with respect to t', *for constant* x, y, z, that is to say, by (8),

$$\frac{dx'}{dt'} = -v, \quad \frac{dy'}{dt'} = 0, \quad \frac{dz'}{dt'} = 0,$$

and this is the vector $-\mathbf{v}$. In exactly the same way, the velocity of S' relative to S is the vector whose components are the derivatives of x, y, z with respect to t, *for constant* x', y', z', *i.e.*, again by (8),

$$\frac{dx}{dt} = v, \quad \frac{dy}{dt} = 0, \quad \frac{dz}{dt} = 0.$$

as we can, $\alpha(v) = 1 \quad for\ 0 \leq v < c$

$\qquad\qquad\quad = i \quad\ \ for\ v > c$

And use $\alpha = \mathrm{i}$ in Silberstein's (9) above and consistently with this also give

$ds^2 = -ds'^2$ which gives GLT for v>c

Note that for GLT the inverse change signs

first, and α in the second set of formulae. As a matter of fact, there is a *physical* reciprocity anyhow, *i.e.* for any $\alpha = \alpha(v)$, subjected to the condition $\alpha(0) = 1$. For the conditions imposed upon the time-labellings in S and in S', in order to make them self-consistent, will continue to be satisfied when all values of time and coordinates, in S or in S', have been multiplied by a common factor; α^{-1} in one, and α in the other case may be thrown back upon the choice of the units of measurement. Thus, the choice of α being a matter of indifference, we *may* take $\alpha = 1$. But, if not content with the physical, we require also a formal reciprocity, then we *have* to write

$$\alpha^{-1} = \alpha, \quad i.e.\ \alpha^2 = 1.$$

But $\alpha(0) = 1$. Thus, if $\alpha(v)$ is to be continuous, $\alpha = +1$.*

In this way we obtain the formulae of what is universally called the **Lorentz transformation**,

$$\left.\begin{aligned} x' &= \gamma(x - vt); \quad y' = y; \quad z' = z \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right), \end{aligned}\right\} \tag{10}$$

already met with in Chap. III. But here, as can be judged from the whole line of reasoning, the meaning and the rôle of this transformation are essentially different from what they were in Lorentz's theory, based as it was on the assumption of a privileged system of reference, the aether.

Let us write also the inverse transformation

$$\left.\begin{aligned} x &= \gamma(x' + vt'); \quad y = y'; \quad z = z' \\ t &= \gamma\left(t' + \frac{v}{c^2}x'\right). \end{aligned}\right\} \tag{10'}$$

The above postulate I., or the Principle of Relativity, may now be expressed in the concise and more definite form:

I^a. *The laws of physical phenomena, or rather their mathematical expressions, are* **invariant** *with respect to the Lorentz transformation.*†

* With regard to Einstein's own treatment of this subject, and also that adopted in Laue's book, see **Note 1** at the end of the present chapter.

† Some authors employ in this connexion the mathematically sanctioned term *covariant*, instead of invariant. But it will be convenient to reserve 'covariant' for another use, namely to denote that two groups of magnitudes are equally transformed.