AN INTRODUCTION TO SIX DIMENSIONAL RELATIVITY

Prediction of dark matter and apparent superluminal velocities

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This report outlines the possibility that the motions of particles and observers, and the description of electromagnetic fields, can take place in a framework of a six-dimensional spacetime. This six-dimensional spacetime comprises three dimensions of space and three of time. Attention will mainly be confined to obtaining the six-dimensional equivalent of the usual four-dimensional description of special relativity. The theory in six dimensions will be called *six-theory*, and the usual four-dimensional description will be called *four-theory.* Proofs are not given in this account – they can be found in the references listed at the end of the report.

We can get on very well by using a four dimensional spacetime in our everyday lives, using only the simplest of theories in order to describe our experiences. For example, since we move around at speeds which are very much less than the speed of light, we do not notice relativistic effects such as time dilation and length contraction. In this case, simple Galilean spacetime transformations involving a universal time are perfectly adequate for our purpose. Again, the weak gravitational fields which affect our everyday lives are adequately explained using simple Newtonian gravitation theory. However, a description of the behaviour of fast moving particles indicate that the simple Galilean transformations must be replaced by the Lorentz transformations of special relativity, in which space and time are inextricably linked, and in which time is not universal. Similarly, the description of strong gravitational fields must be given by the theory of general relativity which is based on curved spacetime, rather than by the simple Newtonian theory. Taking these stages further, a five dimensional relativity proposed by Kaluza [17] and Klein [18] was introduced in order that gravitational and electromagnetic effects be unified – this theory involves the introduction of an extra space dimension. More recently, many more dimensions have been introduced in string theory.

In short, we use an underlying spacetime structure which is as simple as possible in order to describe the physical processes in which we are interested. We recognise that the structure we employ is only one of an increasingly more complex hierarchy of structures, and there is no reason to believe that there is not an infinite hierarchy leading to a description involving perhaps an increasing number of dimensions.

In describing six-theory, we *do not assert* that spacetime is six-dimensional, only that it can be one element (and a fairly naive one at that) lying at the lower end of this hierarchy. It merely means, for example, that the physical processes of vanishing and the observation of apparent superluminal speeds which are predicted in six-theory is best described by this particular element of the hierarchy. Needless to say, the physical interpretation of three time dimensions – how we measure them and why we haven't noticed them – is not straightforward.

Status of the assumptions in six-theory

We do not assert that time exists in a three-dimensional form, but we investigate the consequences of it doing so. The theory that will be developed is simple, and will be based on a minimum number of assumptions; it does not have the sophistication of higher order theories. If it predicts consequences which are directly contradicted by physical reality, then it must be discarded. Although the theory involves motion in different time directions, it does not predict the more lurid aspects of what is commonly regarded as "time travel". Again, one criticism levelled at the idea of motion in different time dimensions is based on the dramatic possibility of going back to kill your maternal grandmother before she had a chance to give birth to your mother. If the theory proposed here did predict that possibility, then the theory would rightly have to be discarded. However, it is demonstrated later that, although it may be possible to go back to encounter your grandmother, you certainly couldn't kill her.

The six-theory presented here must be useful and consistent. It must

1. contain the four-theory based on a one-dimensional time as a special case;
2. explain why motion in the extra time dimensions is not a perceived everyday occurence;
3. predict new and testable physical phenomena.

It is not assumed that there is a preferred time direction. Because time appears to be one dimensional in our everyday lives, we do not assume that this direction is especially singled out. Each observer will have his or her own time direction, and it may be that everyday communication is possible only with other observers whose time directions are in some sense coordinated with our own. For example, the theory shows that objects can be seen to vanish from the sight of an observer if the time paths of these objects differ from that of the observer, and if other conditions are satisfied. Hence it may be that the reality we experience is brought about because all of the objects we experience have time tracks which are very nearly parallel to our own, and that objects which have appreciably different time tracks will not be observed by us. It may be that there are other classes of observers and particles whose time lines are coordinated with each other, but not with ours; in this sense, the current fashion for considering "parallel universes" may yet hold some credance except, of course, that in the context of six-theory it would make more sense to talk of "*non*-parallel universes".

Of course, the theory presented here would be toothless if it could not demonstrate how an observer would be able to detect that an object moves in a different time direction to his or her own time direction. The theory shows how an observer can use the transmission to, and reception of reflected light signals from, an object in order to measure the angle between his or her own time direction and that of the object.

1 Overview of six-theory

Certain aspects of four-theory can be generalised into a six-theory description. In particular,

- An observer is able to describe the motion of an object in terms of its relative velocity and its time direction relative to that of the observer. Of course, since velocity is a measure defined as a rate of change relative to a time, we have to specify carefully what this particular time is.

- The nature of an *inertial frame* in six-theory must be carefully considered, and transformation equations between the six spacetime coordinates used by each inertial observer must be derived.

- A description can be given of the phenomenon of a particle vanishing from the sight of an observer if its velocity relative to the observer is adjusted appropriately. It can be shown that if O and O' are two inertial observers who are permanently visible to each other, then there will be at least one particle P which is permanently visible to one of the observers but permanently invisible to the other. This phenomenon of *differential vanishing* could possibly lead to a description of *dark matter* in six-theory.

- It can be shown how the energy of a particle is now a *vector* quantity, and that an enormous quantity of energy must be supplied to an everyday particle in order that its time direction can be changed by an appreciable amount.

- All of the relative velocities described in the first point will be less than the speed of light c. However, it can be shown how particles can be made to behave as though they had travelled with *apparent* superluminal velocities under certain experimental circumstances: particles must leave a source and follow a time path which differs from that of the observer, only to re-unite with that of the observer at a detector. This will necessitate the use of a form of lens action at some point between the source and detector. This lens would be localised in space, but would have an extent in the time subspace. In the circumstances of such an experiment, six-theory predicts a specific relation between the magnitude of the particle energy and the apparent superluminal speed, and information taken from the plot of this relation can be used to calculate the mass of the particle. This relationship could be tested experimentally.

- Particle mechanics and decays can be discussed; it can be shown how some interactions are allowed in six-theory on energetic grounds, which are not allowed in four-theory without recourse to the uncertainty principle of quantum mechanics.

- It can be shown how new electromagnetic fields can be defined.

- It can be shown how the Dirac equation can be extended into six-theory

We must start by discussing the difficult issue of the arrow of time.

1.1 The arrow of time

The concept of the arrow of time is central to the study of a six-dimensional relativity in which time is three-dimensional. More correctly, in the six-dimensional relativity which is described in this text, the concept must be studied as a two part process: (i) that we must allow for a non-zero angle in the time subspace between the time tracks of the observer and an observed object, and (ii) that we can associate directions to these time tracks.

The first process follows naturally if we make the assumption that time is three-dimensional. Without attaching any directions to the time tracks at the moment, the angle between the track of the observer and the observed object can be characterised by a non-obtuse angle whose value will be observer-dependent.

A discussion of the second process – attaching an arrow to the time track of an object – carries with it all of the baggage and angst that has accompanied discussions of time's arrow in four-theory. This discussion has centred on several processes which have gone into a study of time's arrow [22, 24], involving the *psychological* arrow, the *biological* arrow, the *thermodynamic* arrow, the *cosmological* arrow, and the *radiative* arrow. It is widely considered to be the case that all of these arrows point in the same direction.

We can use the entropy change of a system to associate an arrow of time to a system, but we must also be able to attach an arrow of time to entities which seem to have no internal structure. This comes about because, as we shall see, the energy of an entity will be a *vector* quantity whose direction lies in the time subspace. The conservation of energy as a vector in interactions will require that *all* participants in the interaction have their own time direction.

1.2 Inertial frames

The use of inertial frames is essential to the study of special relativity, and these must be discussed before the properties of the spacetime transformations are derived. In order to set the scene for a discussion in six-theory, it is necessary to give a brief account of the situation in four-theory.

Inertial frames in four-theory

In four-theory, each observer will make measurements in three spatial dimensions, and will use a clock which measures a single time coordinate. The definition of an inertial frame makes reference to Newton's first law of motion:

Newton's first law of motion in four-theory: A free particle remains at rest or moves with constant speed in a straight line.

Based on this law, an inertial frame in four-theory is defined in the following way:

Definition (Inertial frame in four-theory). An inertial frame in four-theory is a frame of reference used for measuring the spatial and time coordinates of events such that
(i) rigid spatial scales determine spatial relations which are Euclidean,

(ii) a universal time exists at each point of the frame, and
(iii) Newton's first law of motion holds when velocities are measured using this universal time.

Part (i) of this definition must be qualified by realising that the notion of rigidity is a property which is frame-dependent.

Part (ii) of the definition relates to a universal time. The idea here is that we can imagine clocks at rest at each spatial point of the frame, and that each clock tells the same time and runs at the same rate as all the others. This idea of a universal time is weaker than that envisaged by Newton, who regarded time as being universal for *all* observers; now, we regard the universality of time as a frame-dependent property. In fact, it turns out that, according to the theory of special relativity, time is *not* universal for all observers. The clocks which are at rest at different points of an inertial frame can be synchronised by transmitting light signals to them from a standard clock situated at the spatial origin O of the frame.

Part (iii) of the definition then states that Newton's first law holds when velocities are defined in terms of Euclidean spatial displacements and the universal time which is supposed to exist throughout the frame. It follows that if S is an inertial frame, then any other frame S' is inertial if and only if it is moving relative to S with constant velocity and without rotation.

Inertial frames are central to the description of special relativity. However, they do not exist: they can be realised only approximately, in regions in which gravity is considered negligible.

Experimental determination of v in four-theory

A particle moves in a straight line with a constant speed v in the inertial frame S. For simplicity, we will consider the special case in which the particle is moving radially relative to the observer who is at rest at the spatial origin O of S. The situation is shown in Figure 1, in which the worldline DD' of the particle has slope v in the $t-x$ plane. It starts at a distance $x = x_0$ at time $t = 0$. A light pulse is sent from the observer at event A, reaches the particle at the event D, and is then immediately reflected back to reach the observer at the event B. Let the times T_A and T_B correspond to the events A and B respectively. Then the pulse can be characterised by the two quantities

$$T \equiv \frac{1}{2}(T_A + T_B) \qquad \text{and} \qquad \Delta T \equiv \frac{1}{2}(T_B - T_A)$$

where T is the *mid-time* of the pulse, and ΔT is the *half-width* of the pulse. Both of these quantities can be calculated once the times T_A and T_B have been recorded. Then the pulse will reach the particle at the time T, and the distance ED of the particle from the observer at this time will be $c\Delta T$. Hence it follows that

$$x_0 + vT = c\Delta T. \tag{1}$$

Note that there are two unknown quantities x_0 and v in this equation, and hence we must make measurements on at least two such pulses. Applying this

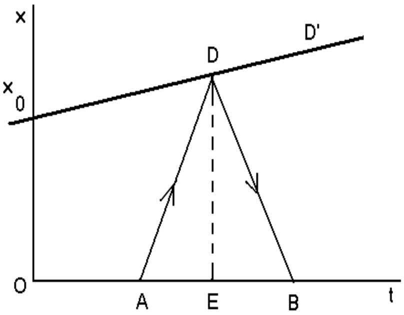

Figure 1: The worldline DD' of a particle moving with speed v in the $t - x$ plane of the inertial frame S. The slope of this line is v, and it passes through the point given by $t = 0$ and $x = x_0$. A light pulse is sent from the observer at the event A, received at the particle at the event D, and immediately reflected back to reach the observer at the event B. The observer records the times of transmission and reception as T_A and T_B respectively.

formula to two pulses whose measurements are characterised by suffices 1 and 2, we have

$$\begin{aligned} \text{pulse 1} \quad &: \quad x_0 + vT_1 = c\Delta T_1 \\ \text{pulse 2} \quad &: \quad x_0 + vT_2 = c\Delta T_2. \end{aligned}$$

On eliminating the quantity x_0 between these two equations, we eventually find

$$\frac{v}{c} = \frac{\Delta T_2 - \Delta T_1}{T_2 - T_1}. \tag{2}$$

Hence by measuring the transmission and reception times T_A and T_B for each pulse, the speed v of the particle can be determined by the observer through Equation (2). In particular, if $x_0 = 0$, then this result becomes

$$\frac{v}{c} = \frac{\Delta T}{T}, \tag{3}$$

and the condition $x_0 = 0$ ensures that the quantity $\Delta T/T$ is independent of which pulse is used.

In the more general case in which the motion is not necessarily radial, then further pulses must be recorded in order to provide further equations from which the extra offset factors can be determined.

Inertial frames in six-theory

The definition of an inertial frame in four-theory must be modified in six-theory, since time is now described by three independent coordinates. There is clearly no problem in retaining part (i) of the definition when a three dimensional time is used. The main change must be made to part (ii), and this change will be accomplished in the following way. There will be three standard clocks at the spatial origin O of the inertial frame S, one for each time direction. At any other spatial point in the frame, there will also be three clocks, one for each time direction. Each of these clocks is then synchronised with the corresponding clock at O by sending appropriate pulses of light. Time displacements obey Euclidean relations in the time subspace, and universal time displacements will then exist in the frame. Part (iii) is considered to hold when velocities are measured relative to this universal time; of course, we must now describe carefully how the velocity of a particle is defined in terms of a particular time direction which is associated with the particle.

Since displacements can now be made in different time directions, Newton's first law of motion must be modified. It will be replaced with

Newton's first law of motion in six-theory: A free particle remains at rest or moves with constant speed in a straight line in space, and moves in a straight line in time.

Based on this modified form of the law, an inertial frame in six-theory is defined in the following way:

Definition (Inertial frame in six-theory). An inertial frame in six-theory is a frame of reference used for measuring the spatial and time coordinates of events such that
(i) rigid spatial scales determine spatial relations which are Euclidean,
(ii) a universal time exists in each time direction at each point of the frame, and time relations are Euclidean,
(iii) Newton's first law of motion holds when the velocity of each particle is measured using a universal time along a time track which is associated with the particle.

Experimental determination of v and time direction in six-theory

The process of determining the speed and time direction of a particle in the inertial frame of an observer is necessarily more complicated than in the four-theory case. But the process can again be carried out by the observer sending light signals to, and receiving reflections from, the particle.

1.3 Postulates of six-theory

The following list of postulates will form the basis of our six-theory.

1. Time is three dimensional.

2. The worldline of any particle is *directed*.

   [Note: the projection of the worldline of the particle into the time subspace is directed by attaching an arrow of time, and this direction is then projected upwards onto the worldline.]

3. There exists a limiting speed c such that if a particle is seen to be travelling with speed c in one inertial frame then it will be seen to be travelling with speed c in any other inertial frame.

   [Notes: (i) experiment identifies the value of $c = 2.998 \times 10^8 \text{ms}^{-1}$ to be the speed of light in a vacuum, (ii) speed is determined as the rate of change of distance with a time parameter; in the framework of a three-dimensional time, this time parameter will be measured along the projection of the particle's worldline into the time subspace.]

4. Einstein's special principle of relativity: all inertial frames are equivalent for the formulation of all physical laws.

1.4 From six-theory to four-theory: the four-theory limit

Generally speaking, four-theory involves the use of 4×4 matrices and four-component vectors, while six-theory involves the use of 6×6 matrices and six-component vectors. Six-theory will also make use of vector displacements in the time subspace, which is now three-dimensional. Hence many of the normal scalar equations of four-theory will be replaced by vector equations in six-theory. Further, results will often be obtained which explicitly contain angles between time vectors. If our six-theory is to be viable, it must contain four-theory as a special case.

The *four-theory limit* enables the standard results of four-theory to be obtained from those of six-theory. In practical terms, we will see that this limit can be obtained when

- the time vectors of all particles and observers are taken to be parallel,
- all time angles are put equal to zero, or are taken to zero in a limiting process.

Clearly, these two limits are equivalent, but each is useful in its own context. The first of these two limits is useful in transforming vector time equations into scalar equations: for example, in transforming the vector mass-energy relation of six-theory into the corresponding scalar equation of four-theory. The second of these two limits is more appropriate for use in expressions in which time angles appear explicitly: for example, in obtaining the Lorentz transformation equations of four-theory from those of six-theory in which time angles appear explicitly.

2 Summary of the results

We now present a brief summary of our approach to six-dimensional relativity, and of some of the important results which will be derived in subsequent chapters. Reference will be made in the following sections to the notions of 6-vectors, in particular to the 6-momenta and 6-velocities of particles These concepts are defined as natural extensions of the 4-vectors of four-theory.

2.1 Space and time transformations in six-theory

The basis of six-theory is a description of the spacetime transformations, but first the corresponding transformations in four-theory will be described. The superscript T will denote the transpose of a matrix throughout the work.

Transformations in four-theory

Let S and S' be two inertial frames moving with relative velocity $\mathbf{v}$ in S and $\mathbf{v}'$ in S', and let the coordinates of an event be described by $X \equiv (x, y, z, ct)^T = (\mathbf{r}, ct)^T$ in frame S and $X' \equiv (x', y', z', ct')^T = (\mathbf{r}', ct')^T$ in frame S'. The linear transformation equations are of the form

$$X' = \Lambda_{(4)} X + X_0$$

where $\Lambda_{(4)}$ is a 4×4 matrix whose elements will be functions of the velocities $\mathbf{v}$ and $\mathbf{v}'$, but are independent of the space and time coordinates. The quantity X_0 is a constant four-component vector. The requirement of light-speed invariance which leads to the result

$$-d\mathbf{r}'^2 + c^2 dt'^2 = -d\mathbf{r}^2 + c^2 dt^2$$

will mean that

$$\Lambda_{(4)}^T G_{(4)} \Lambda_{(4)} = G_{(4)}$$

where $G_{(4)}$ is a 4×4 diagonal matrix $G_{(4)} = diag\{-1, -1, -1, +1\}$. At its simplest (for frames which are in standard configuration which will be described more fully shortly), these transformation equations take the form

$$\left.\begin{array}{rcl} x' &=& \gamma_v (x - vt) \\ y' &=& y \\ z' &=& z \\ t' &=& \gamma_v \left(t - \frac{v}{c^2} x\right) \\ \text{where} \qquad \gamma_v &\equiv& \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \end{array}\right\}, \tag{4}$$

with $|\mathbf{v}'| = |\mathbf{v}|$. These are the *special Lorentz transformations.*

Transformations in six-theory

The situation regarding the spacetime transformations in the six-dimensional case, in which time is three-dimensional, is described as follows [1, 2, 3, 4, 5, 10, 12, 14]. In the inertial frame S, an event will now be located at the space and time points given by

$$\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (x, y, z)^T \qquad \text{and} \qquad \mathbf{t} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = (t_1, t_2, t_3)^T$$

respectively. The position in S of a particle in spacetime will be specified by these six coordinates, and its worldline will now have a direction of motion in time as well as in space. In particular, in specifying the transformation equations between two observers, we must now specify the time directions, as well as the velocities, in each frame of the other's spatial origin. This is done as follows:

- As seen in frame S, the spatial origin $O'$of S' will have a time direction along the unit vector $\boldsymbol{\alpha}_{O'}$. It will have a velocity $\mathbf{v}_{O'} = d\mathbf{r}/dt_{O'}$, where $dt_{O'}$ is measured along the direction $\boldsymbol{\alpha}_{O'}$.
- As seen in frame S', the spatial origin O' of S' will have a time direction along the unit vector $\boldsymbol{\alpha}'_{O'}$. Its velocity in this frame is $\mathbf{v}'_{O'} = \mathbf{0}$.
- As seen in frame S', the spatial origin O of S will have a time direction along the unit vector $\boldsymbol{\alpha}'_{O}$. It will have a velocity $\mathbf{v}'_O = d\mathbf{r}'/dt'_O$, where dt'_O is measured along the direction $\boldsymbol{\alpha}'_O$.
- As seen in frame S, the spatial origin O of S will have a time direction along the unit vector $\boldsymbol{\alpha}_O$. Its velocity in this frame is $\mathbf{v}_O = \mathbf{0}$.

Since there are only two velocities in this specification which are not generally zero, there will be no confusion in writing $\mathbf{v} \equiv \mathbf{v}_{O'}$ and $\mathbf{v}' \equiv \mathbf{v}'_O$.

Writing $X \equiv (\mathbf{r}, c\mathbf{t})^T$ and $X' \equiv (\mathbf{r}', c\mathbf{t}')^T$ for the spacetime coordinates of an event in frames S and S' respectively, the transformation equations will take the form of a linear matrix equation

$$X' = \Lambda X + X_0 \tag{5}$$

where Λ is a 6×6 matrix. Its elements will be functions of the velocities $\mathbf{v}$ and $\mathbf{v}'$, and of the unit time vectors $\boldsymbol{\alpha}_{O'}$, $\boldsymbol{\alpha}'_{O'}$, $\boldsymbol{\alpha}'_O$ and $\boldsymbol{\alpha}_O$. but are independent of the space and time coordinates. The quantity X_0 is a constant six-component vector. The requirement that

$$-d\mathbf{r}'^2 + c^2 d\mathbf{t}'^2 = -d\mathbf{r}^2 + c^2 d\mathbf{t}^2. \tag{6}$$

will mean that

$$\Lambda^T G \Lambda = G \tag{7}$$

where G is a 6×6 diagonal matrix $G = diag\{-1, -1, -1, +1, +1, +1\}$. (In line with our very short introduction to the situation in four-theory, we should really denote this matrix by $G_{(6)}$, but we will only be dealing with this matrix from here on, so we may dispense with the suffix). More specifically, the matrix Λ can be written in the form

$$\Lambda = \begin{pmatrix} \mathbf{A} & \mathbf{P} \\ \mathbf{Q} & \mathbf{R} \end{pmatrix} \tag{8}$$

$$\Lambda^{-1} = G\Lambda^T G = \begin{pmatrix} \mathbf{A}^T & -\mathbf{Q}^T \\ -\mathbf{P}^T & \mathbf{R}^T \end{pmatrix} \tag{9}$$

where the matrices $\mathbf{A}$, $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ are all 3×3 matrices. These matrices will be functions of the velocities $\mathbf{v}$ and $\mathbf{v}'$, and of the unit time vectors $\boldsymbol{\alpha}_{O'}$, $\boldsymbol{\alpha}'_{O'}$, $\boldsymbol{\alpha}'_O$ and $\boldsymbol{\alpha}_O$.

In order to make the resulting equations easier to work with, it is possible to take a form of *standard configuration* in six-theory: this will not be as straightforward as in the four-theory version. In this configuration, we take the corresponding spatial axes to be parallel, with relative motion along the parallel x and x' axes. The position in the time subspace is not so simple: we are not at liberty to say that the unit time vectors of the spatial origins are parallel. The simplest specification is to take the time vector $\boldsymbol{\alpha}_O$ parallel to the t_1 axis, the time vector $\boldsymbol{\alpha}'_{O'}$ parallel to the t'_1 axis, the time vector $\boldsymbol{\alpha}_{O'}$ in the $t_1 - t_2$ plane, and the time vector $\boldsymbol{\alpha}'_O$ in the $t'_1 - t'_2$ plane. Specifically, we will take

$$\begin{array}{lrcl} \text{In frame } S: & \mathbf{v} & = & (v,0,0)^T \\ & \boldsymbol{\alpha}_O & = & (1,0,0)^T \\ & \boldsymbol{\alpha}_{O'} & = & (\cos\theta, \sin\theta, 0)^T \\ \text{In frame } S': & \mathbf{v}' & = & (-v',0,0)^T \\ & \boldsymbol{\alpha}'_{O'} & = & (1,0,0)^T \\ & \boldsymbol{\alpha}'_O & = & (\cos\theta', \sin\theta', 0)^T, \end{array}$$

where θ and θ' are angles in the time subspaces of S and S' respectively. It can be shown that

$$\gamma_v \boldsymbol{\alpha}_O . \boldsymbol{\alpha}_{O'} = \gamma_{v'} \boldsymbol{\alpha}'_{O'} . \boldsymbol{\alpha}'_O \,, \qquad \text{or} \qquad \gamma_v \cos\theta = \gamma_{v'} \cos\theta' \tag{10}$$

where

$$\gamma_v \equiv \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} .$$

The matrix Λ will be shown to be functions of the quantities v, v', θ and θ'.

An important part of our six-theory formulation is to demonstrate how we can experimentally determine the values of v and θ for a particle moving in an inertial frame S. It is possible to show how these values can be determined by bouncing several light pulses off the particle and measuring the transmission and reception times for each pulse. This is similar to the process of determining v alone in four-theory, but in six-theory we must also take timings on clocks which have different time directions.

The four-theory limit can be obtained by putting $\sin\theta = \sin\theta' = 0$ and $\cos\theta = \cos\theta' = 1$ in all of the elements, yielding the result

$$\Lambda = \begin{pmatrix} \gamma_v & 0 & 0 & -\gamma_v \frac{v}{c} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\gamma_v \frac{v}{c} & 0 & 0 & \gamma_v & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \tag{11}$$

Equation (10) will also reduce to the result

$$\gamma_v = \gamma_{v'}, \quad \text{or} \quad |\mathbf{v}| = |\mathbf{v}'|, \tag{12}$$

which is the correct result in four-theory.

2.2 Vanishing

One of the most important and dramatic predictions of our six-theory is that an object may disappear from the sight of an observer if certain conditions are met [13, 15, 16]. This phenomenon depends on the parameters of the motion, in both space and time, of the particle relative to the observer: there will be other observers for whom the particle will not disappear.

Of course, this phenomenon is not predicted in four-theory. In that theory, the persistence of a particle is governed by the interaction of its worldline with the light-surface of the observer, and light from the event at the spacetime point $(x, y, z, ct)^T$ will reach the observer at the spacetime point $(x_0, y_0, z_0, ct_0)^T$ where

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = c^2(t - t_0)^2. \tag{13}$$

If light from a particle reaches the observer at the event $(x_0, y_0, z_0, ct_0)^T$, it is easily demonstrated that, since the particle is moving relative to the observer with a speed less than c, then light from it will continue to reach the observer (unless, of course, the particle is annihilated or light from it is blocked by other bodies).

The situation is more complicated in six-theory. In this case, light from any event which occurs at the spacetime point $(x, y, z, ct_1, ct_2, ct_3)^T$ will reach the observer at the spacetime point $(x_0, y_0, z_0, ct_{10}, ct_{20}, ct_{30})^T$ where

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = c^2\left((t_1 - t_{10})^2 + (t_2 - t_{20})^2 + (t_3 - t_{30})^2\right). \tag{14}$$

In six-theory, it can now be proved that persistance of sight of a particle by an observer is not guaranteed, and a particle can effectively vanish from the sight of an observer, even though the particle is neither annihilated nor has its light blocked by other matter. Specifically, suppose that the particle is moving with a constant velocity $\mathbf{v}$ relative to an observer, and the time angle between the observer's time direction $\boldsymbol{\alpha}_O$ and the particle's time direction $\boldsymbol{\beta}$ is θ. Then it can be shown that

- If the *vanishing condition*

  $$|\mathbf{v}| < c|\sin\theta|, \qquad \text{or} \qquad \gamma_{|\mathbf{v}|}\cos\theta < 1 \tag{15}$$

  holds, then there may be times τ_3 and τ_4 for the observer such that the particle will be visible to the observer in the time interval (τ_3, τ_4), but will not be visible at any time up to τ_3 and after time τ_4. The particle will effectively disappear from the sight of the observer after time τ_4. Equation (15) is never satisfied in four-theory.

- If the condition

$$|\mathbf{v}| \geq c|\sin\theta|, \qquad \text{or} \qquad \gamma_{|\mathbf{v}|}\cos\theta \geq 1 \tag{16}$$

  holds, then the particle will remain visible to the observer up to a time τ_1 and for ever after a time τ_2 on the observer's time path. It may become invisible in the time interval (τ_1, τ_2) if $\tau_1 < \tau_2$. This is the situation which holds in the four-theory limit in which $\tau_1 > \tau_2$.

Notice that the vanishing condition $|\mathbf{v}| < c|\sin\theta|$ is not necessarily a relativistic effect. In fact, it places no minimum restriction on the value of $|\mathbf{v}|$, and it holds even more readily in the non-relativistic case for which $|\mathbf{v}| << c$.

One possibility of this vanishing phenomenon lies in the partial description of *dark matter*. It has been shown [13] that matter which an observer *cannot* see (by virtue of the vanishing condition) is able to influence the behaviour of other matter which the observer *can* see. In this way, visible matter will appear to be acted on by other matter which is not visible. Using six-dimensional special relativity only, we will derive an estimate of the proportion of dark matter in the universe: let the the function $f(v)$ be the distribution function of the speeds v of the matter in the universe. Then the proportion of dark matter in the universe, as predicted by the special six-theory only, will be shown to be $\mu = \langle {\gamma_v}^{-1} \rangle$ where the average $\langle . \rangle$ is taken over the distribution function $f(v)$. For example, a simple decreasing distribution $f(v) = 4/(\pi c){\gamma_v}^{-1}$ will predict a dark matter percentage of 84.9%, a constant distribution $f(v) = 1/c$ will predict a dark matter percentage of 78.5%, while a simple increasing distribution $f(v) = 2/(\pi c)\gamma_v$ will predict a dark matter percentage of 63.7%

2.3 Particle mechanics

It is known that the equations of Newtonian mechanics in four-theory are invariant under a classical Galilean transformation, and that certain assumptions must be relaxed if the equations of mechanics are to be invariant under the Lorentz transformations which are given in Equation (4). In four-theory, the use of plausible assumptions leads to the energy-mass relation $E = mc^2$, which is a scalar equation. This treatment can be extended naturally in six-theory, and leads to the conclusion that energy in now a *vector* quantity [6, 9]. This, in turn, will lead to a plausible reason for the observation that time appears to be one dimensional in our everyday lives.

Particle mechanics in four-theory

We will look first at the situation in four-theory. Suppose that a particle has a rest mass m_0; that is, m_0 is the mass of the particle as measured by an observer for whom the particle is at rest. If the particle then moves with an instantaneous speed w relative to the observer, its *4-velocity* in the frame of the observer is defined to be

$$P^\mu \equiv m_0 \gamma_w (\mathbf{w}, c)^T.$$

When it is assumed that total 4-momentum is conserved at the event of a particle collision or decay, it is found that the mass of the particle in the frame in which it is moving with instantaneous speed w is

$$m = m_0 \gamma_w$$

– that is, its mass increases as its speed increases.

Particle mechanics in six-theory

This treatment is easily extended to six-theory. We must now use the 6-momentum

$$P^\mu \equiv m_0 \gamma_w (\mathbf{w}, c\boldsymbol{\beta})^T.$$

where $\boldsymbol{\beta}$ is the unit time vector of the particle in the frame of the observer. Using arguments which are similar to those used in four-theory, we can now deduce that the energy of the particle is now a vector quantity which is given by

$$\mathbf{E} = mc^2 \boldsymbol{\beta} \tag{17}$$

– that is, the energy of the particle in the inertial frame is now a vector which is directed along the time direction of the particle.

It follows that the equations which represent conservation of energy are now vector equations. In particular, consider the case of a particle with rest mass m_0 at rest with time vector $\boldsymbol{\alpha}_0$ in an inertial frame S. A vector energy $\Delta\mathbf{e}$ is then applied to the particle which makes it move with a speed w and a new time vector $\boldsymbol{\beta}$. Then its time vector has been turned through an angle θ where $\cos\theta = \boldsymbol{\alpha}_0.\boldsymbol{\beta}$, and these quantities will obey the energy conservation condition

$$m_0 c^2 \gamma_w \boldsymbol{\beta} = m_0 c^2 \boldsymbol{\alpha}_0 + \Delta\mathbf{e}. \tag{18}$$

It can be shown that not all applied energies $\Delta\mathbf{e}$ will produce a change, meaning that not all applied energies will be absorbed. Since $\gamma_w \geq 1$, it can be proved from Equation (18) that the applied energy $\Delta\mathbf{e}$ will be absorbed only if it obeys the condition

$$\left| \frac{1}{m_0 c^2} \Delta\mathbf{e} + \boldsymbol{\alpha}_0 \right| \geq 1. \tag{19}$$

There are two main consequences of the vector energy conservation equation which can be described here: the first provides an indication why we do not notice the turning of time vectors in our everyday lives, and the second provides a condition on the applied energy in order to produce vanishing.

- It will follow from Equation (18) that the minimum energy magnitude $|\Delta\mathbf{e}|_{min}$ required to turn the time vector of the particle through an angle θ is given by

  $$|\Delta\mathbf{e}|_{min} = 2m_0 c^2 \sin\frac{1}{2}\theta. \tag{20}$$

  For example, for an object with mass $m_0 = 1$kg , the minimum energy magnitude required to turn the time vector through an angle of 1° can be calculated to be 1.57×10^{15}J: this is roughly equivalent to the energy release of a 375kton explosion, or the equivalent of twenty nine Hirosima atom bombs. The practical aspects of producing this turning are formidable: not only must the object be able to survive the absorption of this energy unscathed, but the energy must also be directed precisely in the time subspace away from the object's initial time direction. This energy is just not available on an everyday basis, and indicates that the objects we see in our everyday lives do not readily change their time directions. On the other hand, the minimum energy magnitude which is required to turn the time vector of an electron through 1° can be calculated as 1.43×10^{-15}J, and hence the energy magnitudes needed to turn the time vectors of quantum particles is greatly reduced.

- Whether or not the vanishing condition of Equation (15) holds for a particle is directly related to the energy applied to the particle. In order to produce a change in the particle's motion, the energy $\Delta\mathbf{e}$ supplied to the particle must satisfy the condition in Equation (19). It can be shown that if the applied energy $\Delta\mathbf{e}$ satisfies the condition

$$-2m_0c^2 < \boldsymbol{\alpha}_0.\Delta\mathbf{e} < 0, \tag{21}$$

then the particle will eventually disappear from the sight of the observer. On the other hand, the long-term persistance of the particle will be guaranteed if either of the conditions

$$\boldsymbol{\alpha}_0.\Delta\mathbf{e} \leq -2m_0c^2 \quad \text{or} \quad 0 \leq \boldsymbol{\alpha}_0.\Delta\mathbf{e} \tag{22}$$

are satisfied.

2.4 Particle interactions and decays

In the context of a three dimensional time, the concepts of "before" and "after" are not well defined, although each observer will have an idea of what these notions mean. The concepts of "before" and "after" now become observer-dependent. These considerations particularly apply to the study of particle collisions and decay, which will be described collectively as particle interactions. The four-theory idea of the 4-momentum being conserved at the event of a particle interaction cannot now be described, in an observer-independent manner, in terms of before and after interaction. However, all observers will agree on the directions of the time vectors of the participating particles, whether they either enter or leave the interaction event. Consequently, we will write down an equation of conservation of 6-momentum in which terms on one side of the equation relate to particles whose time vectors enter the interaction region while those on the other side relate to particles whose time vectors leave the interaction region. Specifically, each particle d which participates in the interaction will be assumed to have a family function $\Phi(d)$ associated with it: this function will be a vector whose elements will include baryon number, lepton number, charge, and other relevant quantities. A collision event C which involves the $m+n$ particles $a_1, a_2 \ldots, a_m, b_1, b_2, \ldots, b_n$ will be denoted by

$$E_C \equiv [a_1 a_2 \cdots a_m, b_1 b_2 \cdots b_n]. \tag{23}$$

The particles $a_i\,(i=1,\ldots m)$ to the left of the separating comma will always denote particles whose time vectors enter the interaction, and the particles $b_j\,(j=1,\ldots n)$ to the right of the separating comma will always denote particles whose time vectors exit the interaction. The ordering of the particles a_i among themselves is not important, and the ordering of the particles b_j among themselves is not important. Let $P^\mu(d)$ denote the 6-momentum of a particle d. Then the interaction E_C given in Equation (23) will be described as *valid* if total 6-momentum and total family function are conserved in the form

$$\sum_{i=1}^{m} P^\mu(a_i) = \sum_{j=1}^{n} P^\mu(b_j) \tag{24}$$

and

$$\sum_{i=1}^{m} \Phi(a_i) = \sum_{j=1}^{n} \Phi(b_j). \tag{25}$$

Since all observers will agree on whether a particle's time vector enters or leaves the interaction event, they will all agree on whether a term should appear on the left hand sides or the right hand sides of these two equations. Now let the 6-velocity of an observer O be $U^\mu_{(O)}$. Then the particle d will be classified as a particle or an antiparticle according to the prescription

$$\begin{array}{lcl} P^\mu(d)U_{(O)\mu} > 0 & : & d \text{ classified as a \textit{particle} by observer } O, \\ P^\mu(d)U_{(O)\mu} < 0 & : & d \text{ classified as an \textit{antiparticle} by observer } O, \\ P^\mu(d)U_{(O)\mu} = 0 & : & d \text{ classified as a \textit{zeroparticle} by observer } O. \end{array} \tag{26}$$

Notice that the class of zeroparticle will be practically empty, since any small deviation of the quantity $P^\mu(d)U_{(O)\mu}$ from zero will push it into either the particle or antiparticle classification. Table 1 shows how the initial/final and particle/antiparticle classification of a particle d is made for any observer O. The signs $+$ and $-$ refer to the sign of $P^\mu(d)U_{(O)\mu}$, and "in" and "out" indicate if the time vector of the particle d is entering or leaving the collision event. Then by taking the inner product of Equation (24) with the 6-velocity $U^\mu_{(O)}$ of the observer O, and re-arranging the terms in such a way that each side contains only positive quantities, the before/after and particle/antiparticle nature of the interaction is specified for that particular observer. Such a re-arrangement of the terms will be described as an observer-dependent *instance* of the interaction. Other observers will have their own, perhaps different, re-arrangements of the terms.

Let $\overline{d}$ denote the antiparticle equivalent of the particle d. Then, for example, the interaction $[a_1, b_1 b_2]$ will have a full list of possible instances as

$$a_1 \quad \to \quad b_1 + b_2$$

Table 1: The initial/final and particle/antiparticle classification of d made for any observer O. The signs $+$ and $-$ refer to the sign of $P^{\mu}(d)U_{(O)\mu}$, and "in" and "out" indicate if the time vector of the particle d is entering or leaving the collision event.

d	initial	final
particle	$+$, in	$+$, out
antiparticle	$-$, out	$-$, in

	in	out
$+$	initial particle	final particle
$-$	final antiparticle	initial antiparticle

$$\begin{aligned} a_1 + \overline{b_1} &\rightarrow b_2 \\ a_1 + \overline{b_2} &\rightarrow b_1 \\ \overline{b_1} + \overline{b_2} &\rightarrow \overline{a_1} \\ \overline{b_1} &\rightarrow \overline{a_1} + b_2 \\ \overline{b_2} &\rightarrow \overline{a_1} + b_1. \end{aligned}$$

It is easily seen that the full list of instances cannot contain those two which have zeros on either side. For our example, this means that the instances

$$\begin{aligned} a_1 + \overline{b_1} + \overline{b_2} &\rightarrow 0 \\ 0 &\rightarrow \overline{a_1} + b_1 + b_2 \end{aligned}$$

will not be valid. Not all other instances will be valid: let the rest masses of the particles a_1, b_1 and b_2 be M_0, m_1 and m_2 respectively. Then it can be shown that the decay $a_1 \rightarrow b_1 + b_2$ is a valid instance only if

$$|{m_1}^2 - {m_2}^2| < {M_0}^2, \tag{27}$$

and if this relation is not satisfied, then there will be no observer for whom the above decay will be valid. In particular, let the particle be at rest in the frame S of the observer for whom the above decay is valid, and let the resulting particles b_1 and b_2 have time directions which make angles θ_1 and θ_2 with that of the initial particle. Then one component of the energy part of the conservation equation (24) will give

$$M_0 = m_1 \gamma_{w_1} \cos\theta_1 + m_2 \gamma_{w_2} \cos\theta_2 \tag{28}$$

where w_1 and w_2 are the speeds of the resulting particles immediately after collision. This result shows that $\gamma_{w_j} \cos\theta_j < M_0/m_j$ for both $j = 1$ and $j = 2$, and hence it can be seen from Equation (15) that the final particle with mass m_j will vanish from the observer's sight if $m_j \geq M_0$. It can also be shown that the resulting particles will either both vanish from, or both persist in, the sight of the observer – they will both vanish if $m_1 + m_2 > M_0$, and will both persist if $m_1 + m_2 \leq M_0$. Note that Equation (28) does *not* imply that $M_0 \leq m_1 + \cdots + m_n$, as it would in four-theory.

It further follows from Equation (27) that a necessary condition for the virtual decay $a_1 \to a_1 + b_2$ to be valid is

$$m_2 < \sqrt{2}M_0.$$

For example, it can be shown for the virtual decay $p \to p + \pi^0$ that the values of the quantity $\gamma \cos\theta$ for the final proton p and pion π^0 are 0.9896 and 0.0720 respectively; both of these values are less than unity, and show that the final particles will vanish. As a second example, the neutron decay $n \to p + e^- + \overline{\nu_e}$ is a valid instance of the interaction $[n\nu_e, pe^-]$ in both four-theory and six-theory. Although the decay $p \to n + e^+ + \nu_e$ is not a valid instance of the interaction $[pe^-, n\nu_e]$ in four-theory, it is possible to find at least one observer for whom the decay is valid in six-theory.

2.5 Apparent superluminal velocities

In late September 2011, neutrinos which were generated at Cern near Geneva, were received in Gran Sasso in Italy at the detectors of OPERA (Oscillation Project with Emulsion tRacking Apparatus).The neutrinos had travelled a distance of approximately 730 km, and appeared to have travelled at speeds greater than the light speed c. It has since emerged that faulty wiring in the measuring apparatus was responsible for erroneous results, and a subsequent repeat of the experiment has produced no superluminal speeds.

However, such apparent superluminal speeds can be predicted by six-theory under certain experimental circumstances: the emitted particle must leave a source and follow a time path which differs from that of the observer, only to re-unite with that of the observer at a detector. This will necessitate the use of a form of lens action at some point between the source and detector. This lens will be localised in space, but will have an extent in the time subspace. In this picture, the particle will leave the source at the event A and travel in a straight spatial line with a constant speed w_A and with its time vector making a constant angle ψ_A relative to that of the observer. It then encounters the lens, and leaves it to travel at a constant speed w_B and time angle ψ_B to meet up with the detector at event B. Since the energy of the particle is now a vector quantity which is directed along the time vector of the particle, and since the time vector of the particle must change in its passage from the source to detector, then the energy of the particle must change by a non-zero quantity $\Delta\mathbf{e}$ at the lens.

We will use the suffix *ob* (denoting *observed*) to denote quantities in our equations which are directly measurable. There is no available measured data, but we will illustrate the calculations by using the data that the OPERA researchers thought they had produced. This data includes the distance X_{ob} over which the particles have travelled, the time T_{light} it would take light to travel that distance, and the time T_{ob} actually taken by the particles. Since the particles appeared to travel faster than the speed of light, they arrive more quickly at the detector than light would have done by an amount $\Delta T_{ob} = T_{light} - T_{ob}$, which has been measured at $(57.8 \pm 7.8) \times 10^{-9}$ s. The observed apparent speed w_{ob} is therefore greater than the light speed c.

This observation of apparent superluminal speeds can be explained in the context of six-theory. In its passage from the source to detector, let the instantaneous speed of the particle be w (which is not greater than c), and let ψ be the instantaneous time angle between the time directions of the particle and the observer. Then one of the main result which can be derived is

$$\frac{1}{w_{ob}} = \left\langle \frac{\cos\psi}{w} \right\rangle$$

where $\langle . \rangle$ denotes an average value over the spatial displacement of the particle. Since $w_{ob} > c$ and $w \leq c$, then there must be at least one non-zero value of the time angle ψ. In the context of a lens action, this means that at least one of the angles ψ_A and ψ_B must reach a required minimum value, which in the case of the data from the OPERA experiment will be calculated as $(6.884 \pm 0.465) \times 10^{-3}$ rad. Again, it is tempting to ask if there is a maximum extent, or aperture, to the lens action in a time direction which is perpendicular to the time direction of the observer, and an extremely crude calculation based on the OPERA data produces a value of 8.40×10^{-6} s, or approximately 2.52 light-kilometers.

In developing the theory, it turns out that there are fewer available equations than unknown quantities. For example, the spatial displacement of the lens between the source and detector will enter the equations, but this displacement is not known. Again, the thickness of the lens is not known, and the energy change $\Delta\mathbf{e}$ at the lens is also unknown. Hence the equations must be supplemented with a number of plausible assumptions. In particular, even though the direction of the particle energy must change at the lens, there is no such requirement on the magnitude e of the energy. If it is assumed that this magnitude does not change, then the theory predicts that the apparent observed superluminal speed w_{ob} is given in terms of this energy by the relation

$$\frac{[{w_{ob}}^2]_e}{c^2} = [\kappa^{-2}] - {m_0}^2 c^4 [\kappa^{-2}] \left(\frac{1}{e^2}\right)$$

where square brackets [.] denote an average over many passages with $[\kappa^{-2}]$ taken as a constant, and m_0 is the rest mass of the particle. Then a plot of $[{w_{ob}}^2]_e/c^2$ against $1/e^2$ should result in a straight line which has a slope $-{m_0}^2 c^4 [\kappa^{-2}]$ and which, if extrapolated backwards, will cross the $[{w_{ob}}^2]_e/c^2$-axis at the value $[\kappa^{-2}]$. A similar relation holds between $[\Delta T_{ob}]_e$ and $1/e^2$ and, providing that the experiments can be repeated sufficiently carefully with different prepared energies e of the particles leaving the source, these relations could provide falsification or partial verification of the theory. The result shows that both w_{ob} and ΔT_{ob} cannot grow indefinitely with e, and both have upper limits.

Apparent superluminal speeds cannot be allowed in four-theory without having to abandon the mass-energy relation, and this fact produced much hand-wringing in the scientific community when the OPERA experimental findings were initially released. But in producing apparent superluminal speeds we can retain the mass-energy relation if we move to six-theory; it seems to me that moving to a vector energy is a smaller price to pay for retaining the mass-energy relation than staying with four-theory and abandoning it altogether.

2.6 Multitemporal ballistics

Two particles are at rest relative to an observer, and all three have the same time vector $\boldsymbol{\alpha}_0$. Can we find an explosive energy $\mathbf{e}$ which, when detonated to propel the particles in opposite directions, allows one or both particles to vanish? Of course we know from the lack of vanishing in four-theory that the directions of both $\mathbf{e}$ and α_0 must be different.

The problem is formulated as follows: two particles 1 and 2 have rest masses m_1 and m_2, and are at rest relative to an observer who is at rest in an inertial frame S. The particles and observer initially have the same time vectors $\boldsymbol{\alpha}_O$. They are packed together with an energy source $\mathbf{e}$ such that the angle in the time subspace between $\mathbf{e}$ and $\boldsymbol{\alpha}_0$ is ψ, so that $\cos\psi = \boldsymbol{\alpha}_0.\mathbf{e}/e$ where $e = |\mathbf{e}|$. In order to be able to make a smoothe transition to the four-theory limit in which $\psi = 0$, we must restrict our considerations to the case $0 \leq \psi < \pi/2$. The energy source is then released to propel the particles so that, for $i = 1, 2$, the ith particle has speed v_i, 3-momentum $\mathbf{p}_i$ and time vector $\boldsymbol{\beta}_i$. The vanishing of particle i will be decided by the condition

$$\Gamma_i \equiv \gamma_{v_i} \boldsymbol{\alpha}.\boldsymbol{\beta}_i < 1 \, ,$$

and we must calculate this value by manipulating the conservation equations for 3-momentum and 3-energy. These equations are

$$\begin{aligned} \mathbf{0} &= \mathbf{p}_1 + \mathbf{p}_2 \, , \\ (m_1 + m_2)c^2\boldsymbol{\alpha}_0 + \mathbf{e} &= \sqrt{{m_1}^2c^4 + |\mathbf{p}_1|^2c^2}\, \boldsymbol{\beta}_1 + \sqrt{{m_2}^2c^4 + |\mathbf{p}_2|^2c^2}\, \boldsymbol{\beta}_2 \, . \end{aligned}$$

from which it follows that $|\mathbf{p}_1| = |\mathbf{p}_2| \equiv p$.

We can now highlight the differences between the four-theory and six-theory versions of these equations. In four-theory, the (scalar) energy equation allows the momentum magnitude p to be calculated exactly, but the directions (equal and opposite) of the 3-momenta are undetermined. In six-theory, these directions are again undetermined, but so is the magnitude p. Further, the left hand side of the above energy equation defines a plane of the initial energies, while the right hand side defines a plane of the final energies. These two planes are not necessarily the same – all we can do is calculate the line of intersection. Hence if ϕ is the angle between these two planes, then the undetermined parameters p and ϕ will appear in the expressions for the quantities Γ_i.

It can be shown that, whatever the initial specification (m_1, m_2, e, ψ) of the problem, it will always be possible to find parameters p and ϕ such that at least one of the particles will vanish.

2.7 Electromagnetism

In six-theory, a nabla operator $\nabla_{\mathbf{t}} = (\partial_{t_1}, \partial_{t_2}, \partial_{t_3})^T$ exists whose derivatives are taken with respect to the three time coordinates, and this allows new electromagnetic fields to be defined [8, 11]. This is in addition to the usual operator $\nabla_{\mathbf{r}} = (\partial_x, \partial_y, \partial_z)^T$ whose derivatives are taken with respect to the three spatial coordinates. In six-theory the vector and scalar potentials of four-theory are replaced by two vector fields $\mathbf{a}$ and $\mathbf{b}$, and the three-component magnetic and electric fields $\mathbf{B}$ and $\mathbf{E}$ of four-theory are replaced by the magnetic field $\mathbf{B}$, a completely new three-component field $\mathbf{W}$ in the time subspace, and a new nine-component field E which replaces the three-component field $\mathbf{E}$. Specifically, we define

$$\mathsf{E} \equiv -\nabla_{\mathbf{r}}\mathbf{b}^T - (\nabla_{\mathbf{t}}\mathbf{a}^T)^T , \qquad \text{or} \qquad E_{ij} \equiv -\frac{\partial b_j}{\partial x_i} - \frac{\partial a_i}{\partial t_j} \quad \text{for } i, j = 1, 2, 3$$

and

$$\mathbf{B} \equiv \nabla_{\mathbf{r}} \times \mathbf{a} \qquad \text{and} \qquad \mathbf{W} \equiv \frac{1}{c^2}\nabla_{\mathbf{t}} \times \mathbf{b}.$$

Equations of motion can be found for a charge moving under the influence of these fields. An electric charge and an observer can have different time vectors, and this has the effect of modifying the effectiveness of the charge. In the same way, a moving charge which has a different time vector to that of the observer will produce modifications in the fields which wash over the observer as the charge passes.

2.8 The Dirac equation

The Dirac equation in four-theory is a first order differential equation for the wave function ψ, and the coefficients of the derivatives with respect to the four spacetime coordinates are matrices which must obey certain anticommutation relations. This allows the function ψ to be the multicomponent quantity which is required for the description of particles with spin 1/2. The anticommutation relations ensure that the Klein-Gordon equation for relativistic particles with no spin is recovered as a special case.

This equation can be extended in a six-dimensional description. The resulting extended Dirac equation will again be a first order differential equation, now with respect to the six spacetime coordinates $\mathbf{r}$ and $\mathbf{t}$. The coefficients of the derivatives will again be matrices, but these will differ from their counterparts of four-theory [2]. Specifically, the spacetime coordinates associated with an inertial frame S will be denoted by x^μ for $\mu = 1, \ldots, 6$, with $x^1 \equiv x$, $x^2 \equiv y$, $x^3 \equiv z$, $x^4 \equiv ct_1$, $x^5 \equiv ct_2$, and $x^6 \equiv ct_3$. The extended Dirac equation for a particle with rest mass m_0 will then be

$$\left(i\xi^\mu \frac{\partial}{\partial x^\mu} - \frac{m_o c}{\hbar} \right) \psi = 0$$

where $\hbar = 1.054 \times 10^{-34}$Js is the reduced Planck constant. The six quantities ξ^μ will be matrices which must satisfy the anticommutation relations

$$\begin{aligned}(\xi^1)^2 = (\xi^2)^2 = (\xi^3)^2 = -(\xi^4)^2 = -(\xi^5)^2 = -(\xi^6)^2 &= -I, \\ \xi^j\xi^k + \xi^k\xi^j &= 0 \quad (j \neq k).\end{aligned}$$

A representation of these matrices can be given in terms of the 8×8 matrices which were originally obtained by Patty and Smalley [23]. The solution ψ will then be an eight-component vector, and it will lead to the concept of spin in the time subspace.

3 Towards a general theory

We will not be concerned with the *general* theory in which spacetime is curved due to the presence of large gravitating masses. However for completeness we describe this general approach. Attepmpts have been made by other workers to obtain exact solutions to the Einstein field equations by introducing higher spatial dimensions [19, 20, 21, 25]; in this section we describe some results which have been derived using extra time dimensions [7].

The spacetime coordinates associated with an inertial frame S will be denoted by x^μ for $\mu = 1, \ldots, 6$, with $x^1 \equiv x$, $x^2 \equiv y$, $x^3 \equiv z$, $x^4 \equiv ct_1$, $x^5 \equiv ct_2$, and $x^6 \equiv ct_3$. In this section only, we will use $g_{\mu\nu}$ to be the metric elements in the curved spacetime, and use $g^{(0)}_{\mu\nu}$ for the corresponding quantities in our special theory. It can be seen from Equation (6) that the invariance of the light speed c in vacuo between inertial frames, together with the requirement of a linear transformation between two inertial frames, means that the quantity

$$ds_0{}^2 \equiv g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -d\mathbf{r}^2 + c^2 d\mathbf{t}^2$$

is invariant in our special theory, where

$$g^{(0)}_{11} = g^{(0)}_{22} = g^{(0)}_{33} = -g^{(0)}_{44} = -g^{(0)}_{55} = -g^{(0)}_{66} = -1 \quad \text{and} \quad g_{ij} = 0 \text{ if } i \neq j\,.$$

In the above expression for $ds_0{}^2$, and throughout this text, we have used the summation convention whereby a summation from 1 to 6 is implied over repeated indices. In the general theory, in which the presence of gravitational fields causes the spacetime to be curved, it is the more general expression

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$$

which is invariant, where now the quantities $g_{\mu\nu}$ will depend on the spacetime coordinates and will reflect the fact that a gravitational field is present.

Weak time-independent fields

Once the Einstein field equations have been solved for the quantities $g_{\mu\nu}$, these can inserted into the geodesic equations which in turn can be solved to give the shape of the path of a test particle which moves in the gravitational field. Of course, all of these solutions can be obtained in principle, if not in practice.

In particular, let it be assumed that (i) the gravitational field is time-independent and weak in the sense that the metric can be written

$$g_{\mu\nu} = g^{(0)}{}_{\mu\nu} + h_{\mu\nu}$$

where $|h_{\mu\nu}| << 1$ and the $h_{\mu\nu}$ are independent of x^4, x^5 and x^6, and (ii) that all velocities are small in the sense that $|dx^i/dt| << c$ for $i = 1, 2, 3$, where dt is the infinitesimal increment of time measured along the projection of the particle's worldline in the time subspace. Then it has been shown that the geodesic equations for a particle with spatial velocity $\mathbf{v}$ and time vector $\boldsymbol{\alpha}$ will reduce to

$$\frac{d\mathbf{v}}{dt} = -\nabla_{\mathbf{r}}\Phi \qquad \text{and} \qquad \frac{d\boldsymbol{\alpha}}{dt} = \mathbf{0}$$

where the potential Φ is given by

$$\Phi \equiv \frac{c^2}{2} \sum_{i,j=1}^{3} \alpha_i \alpha_j \, h_{i+3\, j+3} \, .$$

The first of these equations is the usual Newtonian equation. The second result shows that in weak time-independent gravitational fields in the limit of small velocities, the projection of the worldline in the time subspace of a freely falling particle is a straight line. This will not necessarily be the case in the presence of strong gravitational fields.

A solution of the field equations for spatial spherical symmetry

A spherically symmetric solution of the Einstein field equations in empty space associated with a central mass M has been obtained for the metric

$$ds^2 = -a(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + b(r)\left((dt_1)^2 + (dt_2)^2 + (dt_3)^2\right) \quad (29)$$

where r, θ and ϕ are the usual spatial spherical polar coordinates. Writing $U \equiv 2GM/(c^2 r)$ where G is the gravitational constant, and using the conditions $a \to 1$, $b \to 1$ and $db/dU \to -1$ as $U \to 0$, the functions $a(r)$ and $b(r)$ were found to be given by the implicit solutions

$$\begin{aligned} U &= 2\left(\sqrt{3}-\sqrt{2}\right)^{\sqrt{\frac{3}{2}}} 3^{-\frac{1}{2}} \left(\sqrt{2+a}-\sqrt{3}\right)\left(\sqrt{2+a}+\sqrt{2}\right)^{\sqrt{\frac{3}{2}}} a^{-\frac{1}{2}\left(1+\sqrt{\frac{3}{2}}\right)} \\ &= 6^{-\frac{1}{2}}\left(1-b^{\sqrt{6}}\right) b^{\sqrt{\frac{3}{2}}\left(\sqrt{\frac{3}{2}}-1\right)} \end{aligned}$$

which can be combined to give

$$b^{\sqrt{6}} = \left(\sqrt{3}-\sqrt{2}\right)^2 \left(\sqrt{2+a}+\sqrt{2}\right)^2 a^{-1} \, .$$

I am not convinced that the form of the metric given in Equation (29) is the correct one to take. A more appropriate form to consider may be one which

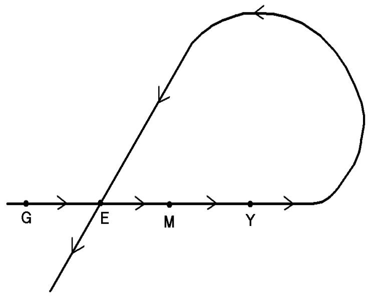

Figure 2: Event G is the birth of your maternal grandmother, M the birth of your mother, Y is your birth, and E is your encounter with your grandmother.

is sperically symmetric in the spatial coordinates r, θ and ϕ as before, but cylindrically symmetric in time. As far as the time coordinates are concerned, let T be measured along the axis of cylindrical symmetry, τ measured from this axis, and ψ the angle measured about the axis. Then a possible metric could be sought whose coefficients could be taken as functions of r and τ. Such a metric should then reduce to the form

$$ds_0{}^2 = -dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + dT^2 + d\tau^2 + \tau^2 d\psi^2$$

as $U \to 0$. The appropriate differential equations satisfied by the coefficients can be easily obtained using suitable algebraic computing packages, although obtaining the solution of these equations is a different matter entirely. The reader who feels suitably challenged is invited to undertake this task. However, as far as this text is concerned, no more will be said on this general theory.

4 Can you go back to kill your grandmother?

Let's face it – one of the main arguments that have been put forward against the idea of movement in three time dimensions is that you could loop back in time to kill your maternal grandmother before she has had the chance to give birth to your mother. In our model, this logical absurdity is not allowed. You would be allowed to go back to have some sort of encounter with her, but you certainly couldn't eliminate her.

Figure 2 shows how such an encounter could take place. The event G represents the birth of your maternal grandmother, event M is the birth of your mother, and Y is your birth. The looped part of the curve represents your time line. The time lines we use in our everyday lives appear to be one dimensional, and this is reflected in the straight line through the points G, E, M, and Y. But since we can now allow curved paths in three time dimensions then, in principle, you could cause your time line to curve back to intersect your grandmother's time line at the encounter E. You can put in extra events relating to the deaths of the various parties if you want. The lines in the figure are time lines, and since we are using a three dimensional time, then the lines are not necessarily all drawn in one plane.

We described briefly in Section 2.4 how the term $E_C \equiv [a_1 a_2 \cdots a_m, b_1 b_2 \cdots b_n]$ describes an interaction between the m particles $a_1, a_2, \ldots, a_m$ whose time vectors enter the interaction region and the n particles $b_1, b_2, \ldots, b_n$ whose time vectors leave the interaction region. Returning to the grandmother question, let Gm and Gc denote the grandmother and grandchild respectively. Then the meeting at the event E would be a valid instance of the interaction

$$E_G \equiv [Gm_{in} Gc_{in}, Gm_{out} Gc_{out}]\,,$$

and the presence of the term Gm_{out} means that the grandmother has emerged unscathed. This is not the same as the interaction $[Gm_{in} Gc_{in}, Gc_{out}]$ in which the grandmother does not emerge unscathed, and it is impossible for both this event and E_G to represent the same event E. In a sense, we might say that the interaction E_G must be "single-valued", just as in quantum mechanics we require that the wave function of a system which is undisturbed by measurement must be single-valued in order to avoid any surrealist happenings. But beware! the event $[Gm_{in} Gc_{in}, Gm_{out}]$ could be a possibility, and so you should be nice to your grandmother when you meet her.

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