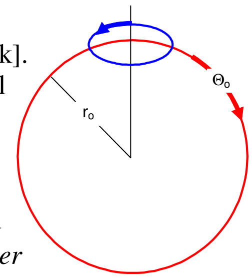
Physics of Galaxies **MID-TERM TEST: ANSWERS**
Time Allowed: 50 minutes **Total marks available 40**
16 November 2011 12:05-12.55 **Physics UG**



1) When a density wave swipes past gas and stars it gives an impulse to them [1 mark]. As a result gas and stars move outward. Because they are given a purely radial impulse, the angular momentum is conserved. $L = mr\Theta = mr_o\Theta_o = const$. Their circular velocity about the centre of the galaxy therefore decreases($\Theta = \Theta_o \frac{r_o}{r} \propto \frac{1}{r}$). Hence, as star/gas moves out, its circular velocity will decrease from the unperturbed value of Θ_o to Θ. It will therefore be moving around the centre of the galaxy *slower* than the other stars which, for a flat rotation curve, all have velocity Θ_o. Relative to these stars, therefore, the perturbed star will appear to be moving backwards. Furthermore, with its reduced velocity, it will now not have sufficient centrifugal force to overcome the gravitational force and so it will tend to drop in towards the centre of the galaxy. As it drops down *below* its original circular orbit at r_o, it will continue to conserve angular momentum and this means that it will now be going *too fast* with respect to stars in circular orbit at this reduced radius and will overtake them. It will also now have too much centrifugal force, so will move back out again. This motion is sketched above. [1 mark + 1 mark for approximate sketch] [1+1+1=**3 marks**]. *Lindblad resonances* are boundaries of the region on an angular frequency versus radius diagram at which resonances occur between the frequency with which a gas cloud meets perturbing effects of the arms (gas angular frequency Ω minus pattern angular frequency Ω_p, i.e. $\Omega-\Omega_p$) [**Note for marker: also accept $\Omega_p-\Omega$**] and its epicyclic frequency κ. It is known that resonance can have a destructive effect on a physical system or an object. (We know that, if we repeatedly "hit" something at its resonant frequency, the oscillations tend to build up and the object may destroy itself.) In the absence of any turbulent velocity, spiral density waves can therefore only exist between the Lindblad resonances, $\Omega - \kappa/m < \Omega_p < \Omega + \kappa/m$ [**3 marks,** do not deduct marks if explanation is reasonable but the inequalities are not stated].

2) Assuming the observer is in the midplane, the pathlength through the dust for a source at Galactic latitude b is x =(h/2)/sin(b). The flux is therefore reduced from F_0 (the value without extinction) to
$F = F_0 \exp(-0.5kh / \sin(b))$. The apparent magnitude is then changed by
$m - m_o = - 2.5 \log_{10}(F/F_o)$
$= - (2.5/\ln 10) \ln(F/F_o)$
$= (1.25/\ln 10)\, kh/\sin(b)$. **[3 marks]** [Note $\log_a x = \log_b x / \log_b a$]
The equation fails because 1/sin(b) goes to infinity when b tends to zero, due to finite radius of the galaxy **[1 mark].**

3) In a trailing spiral, the density wave front moves toward generally *less dense* material (outwards from the centre as it has *positive phase* speed) in which the speed of sound is lower (the speed of sound $u \propto \rho^{(\gamma-1)/2}$ with $\gamma > 1$) **[2 marks]**. It therefore tends to pile up on itself until it produces a shock front because speed of the *wave exceeds the local sound speed*. **[2 marks]**. Since the spiral arms are dominated by hot young stars, therefore, we may conclude that only trailing waves are likely to exist. Only clouds of gas greater than a critical mass M_J could collapse to form stars. This critical mass $M_J \propto \rho^{-1/2}$. If a cloud with nearly the critical mass (or critical size) enters a shock front, this will compress the cloud, increasing its density. If the increase in density is sufficient, the cloud will now be *more* than the critical mass and will collapse in the free-fall (roughly star-formation) time **[2 marks]**. Shock waves thus are capable of causing gas clouds to collapse to form stars in times of the order of 10^6 yr. Since the orbital period of the material about the centre of the galaxy is some 10^8 years, this means that stars form within a few degrees of the shock front ($10^6/10^8$=0.01 – this fraction of full 360^o is about few degrees). The massive O and B stars, which distinguish the arms, stay on the main sequence for only about 10^6 years so that we should expect the arms themselves to be a few degrees wide, as indeed observed **[2 marks]**.

4) Surface brightness of a galaxy *I* is the flux density *dF* emitted per solid angle *dΩ*, *I=dF/dΩ* **[2 marks]**. Magnitudes per solid angle are defined by $\mu = -2.5\log(I/I_0)$. **[2 marks].**

5) The difference is due to the fact that spectroscopic methods rely on *dynamics* of gas/stars. In other words be it viral theorem for ellipticals or Newtonian dynamics for spirals, spectroscopic methods probe the mass of the galaxy using gas/star dynamics. The dynamics occurs in the presence of large amounts of dark matter. *Therefore M/L ratio, measured by spectroscopic methods, includes the contribution from the dark matter* [1 mark]. Photometric methods do not probe dynamics of star/gas and are based on the extrapolation of M/L ratio using Salpeter initial mass function and use of appropriate statistical description. *Therefore photometric methods do not include contribution from the dark matter, only luminous one* [1 mark] – hence the observed difference. **[1+1=2 marks]**

6) If $k < \left(\frac{4\pi G\rho}{u^2}\right)^{1/2}$, then we have from the dispersion relation, $\omega^2 < 0$, so that ω is imaginary which leads to the Jeans instability i.e. exponentially growing solutions. This occurs for the case of Jeans length of $\lambda > \lambda_J = \left(\frac{\pi u^2}{G\rho}\right)^{1/2}$ **[3 marks].** Jeans mass can be obtained from $M = (4\pi/3)R^3\rho$ [1 mark]. If the diameter of the collapsing cloud is *l*=2*R*, one needs to realize that for the possible modes of oscillation, cloud edges need to be nodes of the standing wave because outside the cloud no oscillation can be sustained [1 mark]. A trough or crest of the wave (bounded by cloud edges) is then half the wavelength. Therefore, $l = \lambda_J/2$ and $R = \lambda_J/4$ [1 mark]. Thus, $M_J = (4\pi/3)(\lambda_J/4)^3\rho = \frac{4\pi}{3\cdot 4^3}\left(\frac{\pi u^2}{G\rho}\right)^{3/2}\rho = \frac{\pi^{5/2}}{48}\frac{u^3}{G^{3/2}}\frac{1}{\sqrt{\rho}}$ [1 mark]. **[1+1+1+1= 4 marks].**

Ignoring thermal pressure support effects (zero sound speed) yields $\omega^2 = -4\pi G\rho$, which then results in exponentially growing and decaying solutions $e^{\pm i\omega t} = e^{\pm\sqrt{4\pi G\rho}t} = e^{\pm t/\tau_{ff}}$. Thus the free fall time under gravity alone (without thermal pressure support) is defined as the time over which density of the cloud increases *e* times, i.e. $\tau_{ff} = 1/\sqrt{4\pi G\rho}$ **[3 marks].**

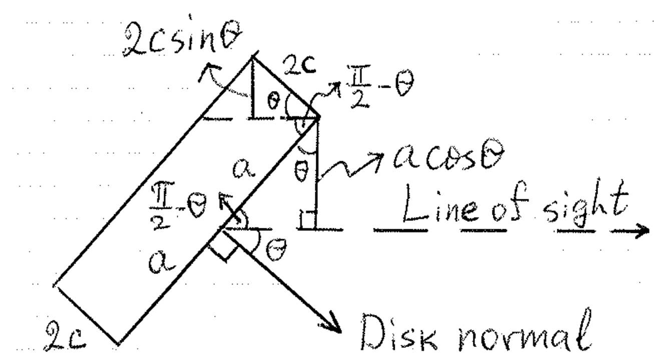
7) Ellipticity is defined as:

$$e = 1 - thickness/width =$$

$$1 - \frac{2a\cos\theta + 2c\sin\theta}{2a} =$$

$$1 - \cos\theta - (c/a)\sin\theta$$

[4 marks]



Simplest argument against the idea that the flattest ellipticals, E7, are disks at an inclination is the value of *c/a* obtained: For E7, *n*=7=10*e*. Thus, $e = 0.7 = 1 - \cos\theta - (c/a)\sin\theta$. Putting in $\theta=\pi/2$ yields *c/a*=0.3 >> than what is observed for spiral galaxies, i.e. 0.01—0.001, typically **[2 marks]**.

[Total marks available 40] **End of Mid-Term Test** **Dr. D. Tsiklauri**