Physics of Galaxies HW: Set 6

Issue Date: 16/11/17 Hand-In Date: 23/11/17
Students should hand in their exercises by 4.00pm on the date given above; exercises will not be accepted for marking later than this. The Course Title, the exercise Set Number and the Student's Name should be stated clearly.

1. The Oort parameters A(r) and B(r) are given, as a function of the distance *r* from the centre of a disc galaxy, by

$$A(r) = +\frac{1}{2}\left[\frac{\Theta(r)}{r} - \frac{d\Theta(r)}{dr}\right];$$
$$B(r) = -\frac{1}{2}\left[\frac{\Theta(r)}{r} + \frac{d\Theta(r)}{dr}\right].$$

In the inner regions of the disc, where the velocity varies approximately linearly with radius, show that the *angular* velocity Ω is approximately constant. **[2 marks]**
Taking its constant value to be Ω_o, show that

$$A(r) \approx 0;$$
$$B(r) \approx -\Omega_o.$$ **[3 marks]**

The epicyclic frequency $\kappa(r)$ is given in terms of the Oort parameters, by

$$\kappa^2(r) = -4\,B(r)\left[A(r) - B(r)\right].$$

Show that, in the inner regions of the galactic disc,

$$\kappa \approx 2\,\Omega_o$$ **[2 marks]**

whereas, in the outer regions, show that

$$\kappa \approx \sqrt{2}\frac{\Theta_o}{r}.$$ **[3 marks]**

2. Are the epicyclic orbits closed in an inertial frame (a) in the inner regions or (b) in the outer regions? **[3 marks]**

3. The Lindblad resonances for an *m*-armed spiral are given by

$$\Omega(r) \pm \frac{\kappa(r)}{m}.$$

Show that, in the outer regions of a two-armed spiral galaxy, these Lindblad resonances are given by

$$\Omega_{\text{inner}}(r) \approx 0.3\frac{\Theta_o}{r};$$
$$\Omega_{\text{outer}}(r) \approx 1.7\frac{\Theta_o}{r}.$$ **[4 marks]**

17 marks in total