**Physics of Galaxies HW: Set 4**

**Issue Date: 26/10/17 Hand-In Date: 02/11/17**

Students should hand in their exercises by **4.00pm** on the date given above; exercises will not be accepted for marking later than this. The **Course Title**, the exercise **Set Number** and the **Student's Name** should be stated clearly.

1. Read relevant chapters and *show* that the impact parameter $b_{\text{strong}}$ in equation $b_{\text{strong}} \sim \frac{2Gm}{v^2}$ makes the gravitational potential energy of a star – in the field of another – equal to its kinetic energy. (Under such a condition, we should expect the gravitational interaction to have a significant effect on the motion of the star. i.e. the system will be *gravitationally relaxed*) **[2 marks]**

2. Use the *linearised* hydrodynamic equations

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \vec{V}_1 = 0; \qquad \frac{\partial \vec{V}_1}{\partial t} = -\frac{\nabla p_1}{\rho_0} - \nabla \varphi_1; \qquad \nabla^2 \varphi_1 \equiv \Delta \varphi_1 = 4\pi G \rho_1; \qquad \nabla p_1 = u^2 \nabla \rho_1;$$

where the symbols have their usual meaning, and linearization of the equations ($f = f_0 + \varepsilon f_1 + \ldots$), assumed that the unperturbed state has the following properties: $\rho_0 = const, \; V_0 = 0, \; p_0 = const$, to derive the following equation for $\rho_1$:

$$\frac{\partial^2 \rho_1}{\partial t^2} - 4\pi G \rho_0 \rho_1 - u^2 \Delta \rho_1 = 0$$ **[8 marks]**.

Further use the latter equation for $\rho_1$ and Fourier ansatz $f = \tilde{f} e^{i(\omega t - kr)}$ to obtain the *dispersion relation* for small perturbations of a self-gravitating gas with non-zero pressure: $\omega^2 = u^2 k^2 - 4\pi G \rho_0$ **[2 marks]**. (Note that this derivation is significantly different from the one in-class).

3. a) Use the dispersion relation $\omega^2 = u^2 k^2 - 4\pi G \rho$ to derive *Jeans length*, $\lambda_J = 2\pi / k_J = \left(\pi u^2 / G\rho\right)^{1/2}$.

**[4 marks]**

b) Derive *Jeans mass*, $M_J = \frac{\pi^{5/2}}{48} \frac{u^3}{G^{3/2}} \frac{1}{\sqrt{\rho}}$.

**[6 marks]**

c) Use the above dispersion relation (in part 3a)) with the thermal pressure effects neglected to derive *the free fall time* and explain its physical meaning.

**[5 marks]**

**27 marks in total**