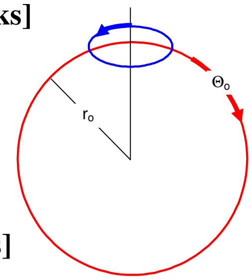
Physics of Galaxies

ANSWERS: SET NUMBER 5

1. The star moves outward under the influence of the impulse. Because the star is given a purely radial impulse, its angular momentum is conserved. $L = mr\Theta = mr^2\Omega = mr_o^2\Omega_o = mr_o\Theta_o$. Its linear velocity about the centre of the galaxy ($\Theta = \Theta_o \frac{r_o}{r} \propto \frac{1}{r}$) therefore decreases and – because the rotation curve is flat – is therefore less than that of the stars amongst which it now finds itself **[2 marks]**. The stars amongst which it now finds itself have the right circular velocity to remain in circular orbit at this distance from the centre of the galaxy. The perturbed star therefore has too little and will have to drop inwards. **[2 marks]**



This motion is called epicyclic motion **[1 mark]** . (For the sketch **[2 marks]**)

[1+2=3 marks]

2.

From Kepler's third law

$$P = \frac{2\pi}{[G(m_1+m_2)]^{1/2}} R^{3/2}.$$

However, $P = 2\pi R/\Theta$. Substituting and rearranging,

$$\Theta = \left[\frac{G(m_1+m_2)}{R}\right]^{1/2}.$$

Differentiating our expression for Θ gives

$$\frac{d\Theta}{dR} = -\frac{1}{2}\frac{[G(m_1+m_2)]^{1/2}}{R^{3/2}} = -\frac{1}{2}\frac{\Theta}{R}.$$

Substituting into Eqs. leads to

$$A = \frac{3}{4}\frac{\Theta_0}{R_0}$$

$$B = -\frac{1}{4}\frac{\Theta_0}{R_0}.$$

Using the Keplerian expressions, we find $A = 20.6\ \mathrm{km\,s^{-1}\,kpc^{-1}}$ and $B = -6.88\ \mathrm{km\,s^{-1}\,kpc^{-1}}$. The values do not agree because they assume that all of the Galaxy's mass resides interior to R_0.

[3 marks] [3 marks] [2 marks] [3 marks] [Total marks available 18]