Physics of Galaxies

ANSWERS: SET NUMBER 3

1. Photometric method depends on knowing the mass-luminosity (M/L) ratio of galaxies. We use observations within our own galaxy to get Galactic values of M/L and assume that we can extrapolate these values to other galaxies. First one calculates $M_{\text{total}} = \int_{M_{\text{low}}}^{M_{\text{high}}} \phi(M) M dM$ using Salpeter initial mass function $\phi(M) = \phi_*(M/M_*)^{-\alpha}$ (with alpha being 2.35). Then use mass-luminosity empirical relation, $L(M) = L_*(M/M_*)^{3.3}$, to calculate $L_{\text{total}} = \int_{M_{\text{low}}}^{M_{\text{high}}} \phi(M) L(M) dM$. This enables to calculate the mass-luminosity ratio, measured in solar values. Once we know that ratio (it turns out to be close to unity) one uses photometric measurements to deduce total luminosity of a desired galaxy, thus finally obtaining its total mass **[4 marks]**.

Step 1: use virial theorem $2T + \Omega \approx 0$;**[1 mark]** Step 2: calculate potential energy of the galaxy using $\Omega = -\alpha GM^2/R$ where alpha is constant of order unity **[1 mark]** Step 3: calculate twice kinetic energy of stars using $2T \approx M\langle v^2 \rangle$ where $\langle v^2 \rangle$ is mean-square velocity **[1 mark]** Step 4 combine above to get $M \approx R\langle v^2 \rangle/(\alpha G)$ and by measuring $\langle v^2 \rangle$ from Doppler shift which results in effective broadening of an absorption line of large collection of stars, the mass M is obtained (size R is known from photometry measurements) **[1 mark] [1+1+1+1=4 marks].** When applying Newtonian dynamics to spiral galaxies one determines circular rotation speed from which mass enclosed within a certain radius can be obtained **[2 marks]**. The method name is spectroscopic [1 mark] and it uses Doppler shift effect of emission or absorption lines [1 mark] **[1+1=2 marks]**.

2.

$$L_{\text{total}} = \int_{M_l}^{M_h} \phi(M) L dM = \int_{M_l}^{M_h} \phi_* \left(\frac{M}{M_*}\right)^{-\alpha} L dM = \int_{M_l}^{M_h} \phi_* \left(\frac{M}{M_*}\right)^{-\alpha} L_* \left(\frac{M}{M_*}\right)^{3.3} dM = \frac{\phi_* L_* \left(M_h^{4.3-\alpha} - M_l^{4.3-\alpha}\right)}{M_*^{3.3-\alpha}(4.3-\alpha)} =$$

$$\frac{\phi_* L_* M_h^{4.3-\alpha}}{M_*^{3.3-\alpha}(4.3-\alpha)} \left(1 - \left(\frac{M_l}{M_h}\right)^{4.3-\alpha}\right) \approx \frac{\phi_* M_* L_*}{(4.3-\alpha)} \left(\frac{M_h}{M_*}\right)^{4.3-\alpha}$$

In the last line we used the fact that from Salpeter IMF $\left(\frac{M_l}{M_h}\right) \approx 0.001 << 1$ and the fact that alpha =2.35<4.3 **[4 marks].** The total mass of the galaxy is prescribed by the low mass stars because Salpeter IMF shows that there are a lot more low mass stars than high mass stars in the distribution. [1 mark] The total luminosity is prescribed by the high mass stars because the mass-luminosity empirical relation, $L(M) = L_*(M/M_*)^{3.3}$, tells us that more massive stars are significantly more luminous (note the power of 3.3!) [1 mark] **[1+1=2 marks]**.

3. Using spectroscopic method elliptical galaxies have mass-luminosity ratios, measured in solar values, in the range of (20--40) *h* [1 mark] whereas the value for spirals is around 10 *h* [1 mark]. The ratio is of the order of unity using the photometric method [1 mark]. The discrepancy is due to the fact that photometric method overlooks contribution from the dark matter because it is not a dynamical method (statistical rather, based on convolution integral of *M* and *L* with the *initial mass function*). Spectroscopic method uses dynamics of stars/gas hence includes gravitational pull contribution from the dark matter [1 mark] **[1+1+1+1=4 marks]**.

4. $(M/M_{sun})/(L/L_{sun})=(M/M_{sun})^{-2.3} = 50^{-2.3} = 0.00012$ for the star **[1 mark]**. Now for average human first work out human luminosity (power output due to metabolism): L_{human} = energy / time = 2400x4200 /(24x60x60) = 116 W. (here 4200 comes from 1kcal=1000cal=4200J – remember calorie is the energy needed to heat 1 gram of water by 1K and specific heat of water is 4200 J $kg^{-1}K^{-1}$). Thus one obtains for a human body $(70/M_{sun})/(116/L_{sun})=0.00012$ – the same as for the above O-type star! **[4 marks].** Now for the human brain: $(0.02\text{x}70/M_{sun})/(0.2\text{x}116/L_{sun})=0.00001165$

yielding $M/M_{sun} = 0.00001165^{-1/2.3} = 140$ (that's about the heaviest O-type star there is) **[2 marks].**

[Total Set 3 marks available 29]