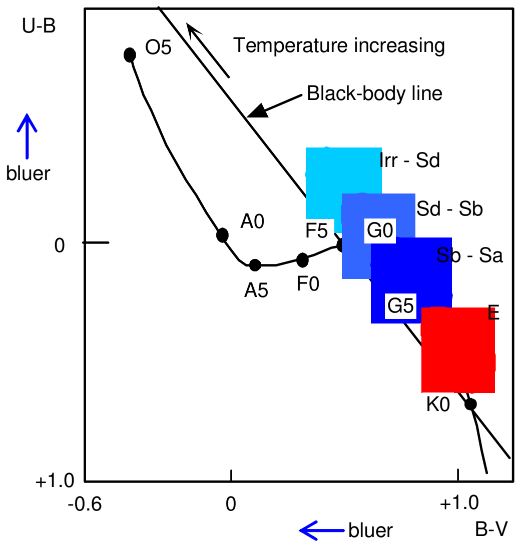
Physics of Galaxies

ANSWERS: SET NUMBER 2

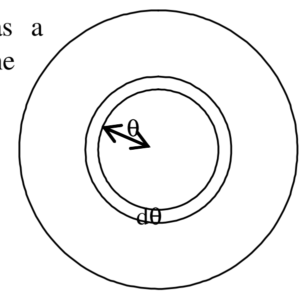
1. *A colour* is a measure of the ratio of the flux densities of a source at two different wavelengths. **[2 marks]**



[See the sketch. Black-body curve and main sequence helpful but not essential.] **[3 marks]**

Hot young stars are blue whilst, on the whole, red stars are old. The figure shows that elliptical galaxies are redder than spirals and that spirals get bluer as one goes along the Hubble sequence. This suggest that ellipticals contain predominantly old stars whilst spirals contain increasingly more young stars as we go from *a* to *d*. **[2 marks]**

2. The surface brightness of a galaxy is its flux density per unit solid angle, as a function of position in the galaxian image. It is a useful physical quantity because it is independent of the distance to a galaxy. **[3 marks]**



3. Because $I(\theta)$ is given only as a function of θ, we can assume that the image of the galaxy has circular symmetry. Consider an annulus of (angular) radius θ and width $d\theta$. The solid angle $d\Omega$ subtended by this annulus is given by

$$d\Omega = 2\pi\theta d\theta \qquad (1.1)$$

and the flux dF coming from it is given by

$$dF = I(\theta)\times 2\pi\theta d\theta\,. \qquad (1.2)$$

The total flux density of the galaxy is therefore given by

$$F = \int_0^\infty I(\theta) 2\pi\theta d\theta\,. \qquad \textbf{[3 marks]} \quad (1.3)$$

4. Substituting for the de Vaucouleurs profile in equation (1.3), we get

$$F = \int_0^\infty I(0)\exp\left[-\left(\frac{\theta}{\theta_o}\right)^{1/4}\right] 2\pi\theta d\theta = 2\pi I(0)\theta_o^2 \int_0^\infty \exp\left[-\left(\frac{\theta}{\theta_o}\right)^{1/4}\right]\left(\frac{\theta}{\theta_o}\right) d\left(\frac{\theta}{\theta_o}\right). \quad (1.4)$$

If we put

$$x = \left(\frac{\theta}{\theta_o}\right)^{1/4}, \qquad (1.5)$$

equation (1.4) becomes

$$F = 2\pi I(0)\theta_o^2 \int_0^\infty e^{-x} x^4 d\left(x^4\right) = 2\pi I(0)\theta_o^2 \times 4\int_0^\infty e^{-x} x^7\, dx = \pi I(0)\theta_o^2 \times 8 \times 7! \qquad (1.6)$$

using the standard integral (1.4) given in the question. Hence,

$$F = 8!\pi\theta_o^2 I(0) \qquad \textbf{[5 marks]} \quad (1.7)$$

5. From equation (1.3), we have for the fictitious galaxy,

$$F_{\text{fict}} = \int_0^{\infty} I(\theta) 2\pi\theta d\theta = 2\pi I_{\text{o}} \int_0^{\theta_{\max}} \theta d\theta = \pi\theta_{\max}^2 I_{\text{o}} \quad (1.8)$$

If

$$I_{\text{o}} = I(0) \quad \text{and} \quad F_{\text{fict}} = F \quad , \quad (1.9)$$

then we have from equations (1.7) and (1.8)

$$\pi\theta_{\max}^2 = 8!\pi\theta_{\text{o}}^2 \quad (1.10)$$

so that

$$\theta_{\max} = \sqrt{8!}\theta_{\text{o}} \approx 200\theta_{\text{o}} \, . \qquad \textbf{[5 marks]} \quad (1.11)$$

[Total Set 2 marks available 23]