Answer **THREE** questions

The numbers in square brackets at the right-hand edge of the paper indicate the provisional allocation of maximum marks for each subsection of a question.

1. Define the terms *mass number A, atomic number Z* and *binding energy B.*

[3 marks]

Sketch the form of binding energy per nucleon $\overline{B}$ as a function of A for $A \geq 20$ and indicate the value of A where the principal maximum occurs.

[3 marks]

For large values of A, $\overline{B}$ for the most stable nuclei may be approximated by the expression

$$\overline{B} = [9.40 - 7.7\,\text{x}10^{-3} A]\ \text{MeV/nucleon}$$

Use this to estimate the value of A beyond which α-decay becomes possible. You may use the binding energy of the α-particle $B_\alpha = 28.3\text{MeV}$ and assume that

$$B(A,Z) - B(A-4, Z-2) \approx 4\frac{\partial B}{\partial A}$$

[5 marks]

Define the term *magic number.*

[2 marks]

The energy level diagram for the first four levels of a nucleus assuming a simple harmonic oscillator potential are:

———— 2p, 1f

———— 2s, 1d

———— 1p

———— 1s

Explain the notation and work out the magic numbers for this system. How does this differ from observation? What interaction must be added to the Hamiltonian to modify the energy level structure to recover the correct magic number sequence?

[4 marks]

In a realistic potential, the ordering of levels is

$$1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 1f_{7/2}, \ldots$$

where the subscript is the *j*-value. Use this to find the expected ground-state spin and parity for ${}^{39}_{19}K$ in the single-particle shell model.

[3 marks]

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2. State briefly the evidence for the *colour* quantum number obtained from the reaction $e^+e^- \to \text{hadrons}$.

[2 marks]

Consider a scenario where the colour quantum number does *not* exist, so that the overall wavefunction Ψ of a hadron consists of just space and spin parts, i.e.

$$\Psi = \psi_{\text{space}}\psi_{\text{spin}}$$

What would be the resulting multiplet structure of the ground-state baryons composed of u, d and s quarks?

[5 marks]

The reaction $e^+e^- \to \tau^+\tau^-$ is studied using a collider with equal beam energies of 5 Gev. The differential cross-section is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\hbar^2c^2}{4E_{\text{cm}}{}^2}\left(1+\cos^2\theta\right)$$

where E_{cm} is the total centre-of-mass energy and θ is the angle between the incoming e^- and the outgoing τ^-. What is the total cross-section for this reaction in nanobarns?

[3 marks]

If the reaction is recorded for $10^7\,\text{s}$ at a luminosity of $L = 10^{31}\,\text{cm}^{-2}\text{s}^{-1}$, how many events are expected?

[2 marks]

The detector is of cylindrical construction and at increasing radii from the beam line there is a drift chamber, an electromagnetic calorimeter, a hadronic calorimeter and finally muon chambers. Explain briefly the physical processes that underlie the drift chamber and the electromagnetic calorimeter and the main characteristics of these two detectors.

[4 marks]

If in a particular event the τ^- decays via $\tau^- \to \mu^- + \overline{\nu}_\mu + \nu_\tau$ and the τ^+ decays to $\tau^+ \to e^+ + \overline{\nu}_\tau + \nu_e$, what signals would be observed in the various parts of the detector?

[2 marks]

If the detector can only record an event if the $\tau^+\tau^-$ pair makes an angle of at least 30^0 relative to the beam line, what fraction of events will be recorded?

[2 marks]

[***Numerical data***: $\alpha = 1/137$; $\hbar c = 1.97\text{x}10^{-16}\,\text{GeV}\,\text{m}$]

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3. Explain qualitatively why weak interactions appear to be very much weaker than electromagnetic interactions at low energies, but of comparable strength at high energies.

[3 marks]

Draw a Feynman diagram at the quark level for the decay $\Lambda \to p + \pi^-$. If nature were to double the weak coupling constant and decrease the mass of the W boson by a factor of four, what would be the effect on the decay rate $\Gamma(\Lambda \to p + \pi^-)$?

[4 marks]

The cross-section for the production of the Z^0 boson in e^+e^- collisions and its subsequent decay into fermion-antifermion pairs $f\,\bar{f}$ is given by

$$\sigma(e^+e^- \to Z^0 \to f\,\bar{f}) = 12\pi\hbar^2c^2\left[\frac{\Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{(E_{\mathrm{cm}}{}^2 - M_Z{}^2c^4)^2 + M_Z{}^2\Gamma_Z{}^2c^4}\right]$$

where M_Z, Γ_Z are the mass and width of the Z^0, and $\Gamma_{e^+e^-}$ and $\Gamma_{f\bar{f}}$ are the partial widths for decay to e^+e^- and $f\,\bar{f}$ pairs, respectively.

What quark pairs are allowed for the final state $f\,\bar{f}$? Calculate the maximum cross-section in nanobarns for all quark pair production.

[4 marks]

What is observed experimentally when pairs of quarks are produced? Which other fermion pairs can the Z^0 decay to and what is observed experimentally in each of these decays?

[3 marks]

Use the data supplied to show that the measured width of the Z^0 implies that there are only three species of light neutrinos.

[2 marks]

State briefly why it is necessary to introduce a Higgs boson into the Standard Model.

[2 marks]

One way of looking for the Higgs boson H is in the reaction $e^+e^- \to Z^0H$. For a Higgs boson with mass < 120 GeV, the branching ratio for $H \to b\bar{b}$ is predicted to be 85%. Why will looking for b-quarks help distinguish $e^+e^- \to Z^0H$ from the background reaction $e^+e^- \to Z^0Z^0$?

[2 marks]

[***Numerical data***: $\alpha = 1/137$; $\hbar c = 1.97\text{x}10^{-16}\,\mathrm{GeV\,m}$; $M_Z = 91\,\mathrm{GeV/c^2}$; $\Gamma_Z = 2.56\,\mathrm{GeV}$;
$\Gamma_{q\bar{q}} = 302\,\mathrm{MeV}(388\,\mathrm{MeV})$ for quarks with charge Q = 2e/3($Q = -e/3$);
$\Gamma_{\nu_\ell\bar{\nu}_\ell}(\ell = e,\mu,\tau) = 175.5\,\mathrm{MeV}$; $\Gamma_{\ell^+\ell^-}(\ell = e,\mu,\tau) = 88\,\mathrm{MeV}$]

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4. Define the term *prompt neutron* in a fission process. If the fission of a nucleus produces on average n prompt neutrons in the final state and the probability that a newly created neutron itself induces fission is p, show that the number of neutrons present at time t is given by

$$N(t) = N(0)\exp\left[(np-1)t/t_p\right]$$

where t_p is the average time to produce prompt neutrons.

[3 marks]

If, for reasons of safety, reactors are designed so that $(np-1)<0$ always, how is this equation modified in practice so that it is possible to have a sustainable reaction in a practical power reactor?

[2 marks]

On the same diagram, sketch the total cross-section σ_{tot} and the fission cross-section σ_f as functions of energy for neutrons incident on $^{235}_{92}U$. On another diagram sketch the corresponding cross-sections for $^{238}_{92}U$. Comment on the physical features of the two diagrams that effect the behaviour of the nuclei in a reactor, for the three regions $\mathrm{E} < 0.1\ \mathrm{eV}$; $0.1\ \mathrm{eV} < \mathrm{E} < 1\ \mathrm{keV}$; and $\mathrm{E} > 1\ \mathrm{keV}$.

[7 marks]

Use these curves to explain why a chain reaction is not possible in natural uranium even though $n = 2.2$. You may assume that the average energy of a fission neutron in $^{235}_{92}U$ is 2 MeV and that most of the fission neutrons in $^{238}_{92}U$ have energies greater than 1.4 MeV.

[4 marks]

If a chain reaction is not possible in a mass of purely natural uranium, how in practice can natural uranium be used as a fuel in a power reactor?

[4 marks]

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5. Draw *one* lowest-order Feynman diagram for each of the following processes:

(a) production of a $t\bar{t}$ pair in proton-antiproton annihilation involving a single gluon;
(b) interaction of a quark and a gluon to produce a quark and a photon;
(c) lowest order gluon-gluon scattering.

[6 marks]

Inelastic electron-proton scattering at high energies is

$$e^{-}(E,\boldsymbol{pc}) + p(E_P,\boldsymbol{P}_P\boldsymbol{c}) \rightarrow e^{-}(E',\boldsymbol{p'c}) + X(hadrons)$$

Draw the lowest Feynman diagram for this process at the quark level.

[3 marks]

The variable ν is defined by

$$2M\nu \equiv W^2c^2 + Q^2 - M^2c^2$$

where M is the proton mass, W is the invariant mass of the hadrons and

$$Q^2 = (\mathbf{p}-\mathbf{p'})^2 - (E-E')^2/c^2 .$$

Use energy-momentum conservation to show that $\nu = E - E'$ in the rest frame of the proton. Hence show that the variable x defined by

$$x \equiv Q^2/2M\nu$$

lies in the range $0 \le x \le 1$ if the mass of the electron is neglected. State the physical interpretation of x?

[7 marks]

What is meant by the terms *structure function* and *scaling*? Explain briefly why scaling is not exact and the origins of the corrections to it. State *one* fundamental properties of quarks that can be deduced from inelastic lepton-nucleon scattering at high energies.

[4 marks]

[For a particle of total energy E, mass m and momentum $\mathbf{p}$, $E^2 = \mathbf{p}^2c^2 + m^2c^4$]

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