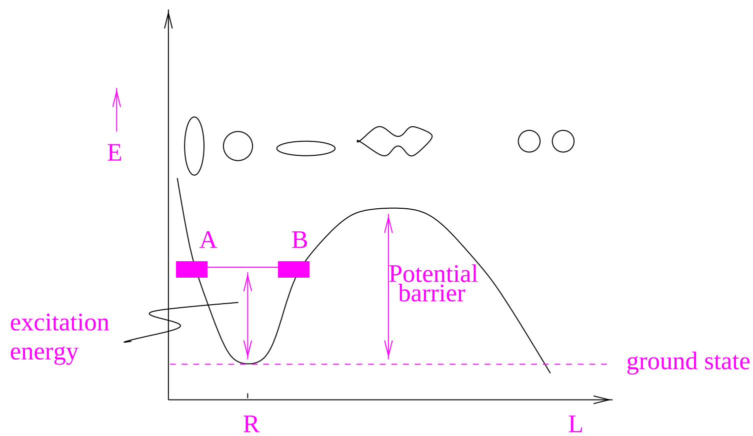
Solutions to Problem Sheet 5

3C24

March 21, 2000

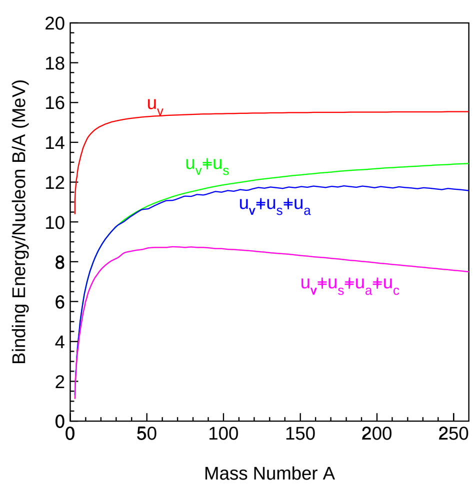
1. i) Draw a diagram to show the stages of fission and accompany it with a graph of the energy necessary for each step. [2]



ii) Two possibilities exist for enabling this to happen: what are they? [2]

The first possibility is spontaneous fission, which usually happens when one of the fragements is much smaller than the other and can successfully tunnel through the potential Coulomb barrier. MOstly, these nuclei are far from the line of stability. Alpha decay is the limit of fission into two maximally different sized fragments. The second possibility is induced fission whereby energy is added to the nucleus in order to bring it over the potential barrier and enable it to break up. The energy is usually in the form of a neutron of sufficient energy.

iii) Sketch a graph of the relative sizes of the terms in the SEMF as a function of A [4]



iv) Using only an order of magnitude calculation, compare the relative energy release in a chemical and a nuclear reaction [4]

Consider a chemical reaction such as occurs in an explosion of TNT. A valence electron will be torn out of its shell and replaced around another atom or compound. The energy emitted is given by

$$E = \frac{e^2}{R} \tag{1}$$

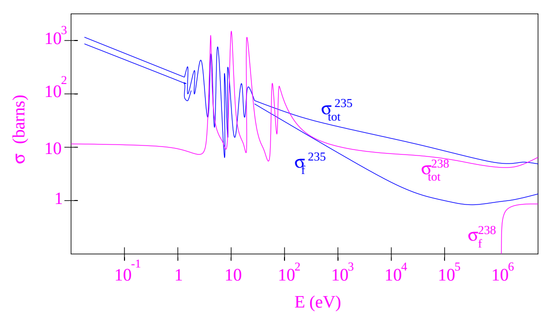
R in the case of the atomic radius is R_{atom}=10^{-10}*m which is* 10^4 *times the size of a Uranium nucleus. The electron can be considered as moving away to infinity from* R_{atom}.

Uranium has 92 protons so $E_{parent} = (92)^2/R_{nucleus}$ *and* $R_{nucleus}$ *is* 10^{-14}*m. After the decay to two equal sized pieces, the total energy of the system is* $2E_{daughter} = 2.1.26.(46)^2/R_{nucleus}$. *The factor of 1.26 comes in because the two daughters now have smaller radii than the parent by a factor* $1/^3\sqrt{2}$. *The difference between the original energy and the final energy is* $3{\cdot}10^{17}$ *(in some units) compared with* $1{\cdot}10^{10}$ *in the case of the chemical reaction. This shows that the nuclear reaction gives out on the order of a whopping* 10^7 *times more energy than a chemical process. In the case of explsoives, 1kg of* $^{235}U \equiv 3 \cdot 10^3$ *TONS of TNT.*

2. i) Why is it necessary for ^{235}U to absorb a neutron before decaying? [2]

In order to push the nucleus over the Coulomb potential barrier, energy must be added to the nucleus. This is done by way of a slow neutron. Once the system is over the potential barrier it will break up.

ii) Sketch the dependence of the neutron capture cross-section on the neutron energy for ^{235}U and ^{238}U [4]



iii) Why is ^{238}U stable? [4]

There is essentially zero probability that a neutron will be absorbed by ^{238}U *unless its energy is above about 1MeV. Of the 75% of neutrons which have enough energy, 25% of them fail to be slowed down to below this 1MeV threshold before being absorbed. Therefore, there is a factor of 0.19 times the number of neutrons emitted in a fission. This average number of neutrons per fission is 2.2 (for all U chains) so the* **effective** *number of neutrons per fission for* ^{235}U *is 2.2 times 0.19* $\approx$ *0.4. Not enough to produce a chain reaction.*

iv) Why is plutonium a more efficient fission fuel than Uranium?[1] Plutonium gives out on average 3 neutrons per fission whereas Uranium gives only 2.2 on average.

3. i) Explain what is meant by critical mass [2]

A critical mass is the smallest mass of a fissile material which can produce and sustain a chain reaction

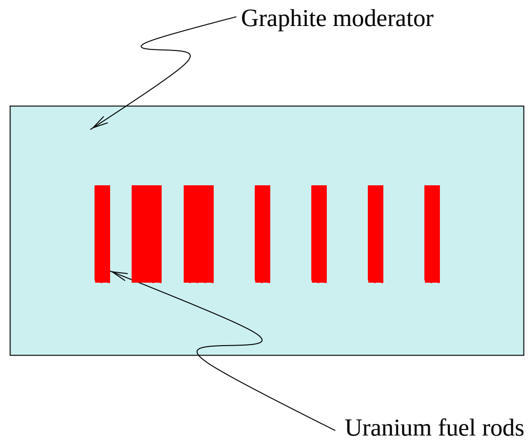
ii) List the effects which affect the size of the critical mass.[6]

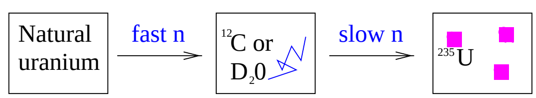
- ∗ *The energy of the secondary neutrons and consequently their fission absorption cross section (depends on the material).[2]*
- ∗ *The number of secondary neutrons per fission.[1]*
- ∗ *The density of fissile material.[1]*
- ∗ *The thermal expansion rate of the fissile material once the reaction has started.[1]*
- ∗ *The use of a tamper can reduce the critical mass in thermonuclear devices.[1]*

iii) In an explosive device, why must the fission proceed rapidly? [2]

Fission must be made to proceed rapidly in order that a sizable amount of the material fissions before too many neutrons are allowed to escape from the thermally expanded and therefore less dense material and the chain reaction is brought to a halt.

4. i) Draw a schematic diagram of a nuclear power reactor.[4]





ii) Explain the considerations which apply when designing a power reactor.[4]

Total number of neutrons produced in the reaction must be ≈1. This must include not only the prompt neutrons emitted in the fission itself, but also the later ones which come from decays of the daughter products. This ensures that a chain reaction is sustained but at a constant rate. The exact rate must be controllable, either by use of a moderator or by the ability to extract the fuel rods from the neutron rich surroundings to reduce the number of neutrons reabsorbed.

iii) Describe the mode of operation of a fast breeder reactor and compare this with that of a thermal reactor. [4]

In the **fast breeder** *reactor fission is induced directly by the fast neutrons and so the Uranium must be enriched to 20% ^{235}U in order to sustain a constant rate of fission. This does not utilize a moderator. Thermal reactors have the advantage of utilizing natural Uranium because the neutrons are slowed down to thermal energies (1eV) such that their capture probability is 84% in the ^{235}U present in the naturally occuring Uranium. The neutrons are slowed down by the* **moderator** *which is usually graphite or heavy water (D_2O).*

iv) What is the important by-product of a fast-breeder reactor and how is it produced?[2]

A fast breeder reactor produces ^{239}P:

$$^{238}U + {}^{1}n \rightarrow {}^{239}_{92}U + \gamma$$
$$^{239}_{92}U \rightarrow {}^{239}_{93}Np + \beta^- + \bar{\nu}_e \;\; T{=}23.5min$$

$$^{239}_{93}Np \rightarrow ^{239}_{94}Pu + \beta^- + \bar{\nu}_e \;\; T=2.35 \; days$$
$$^{239}_{94}Pu \rightarrow ^{235}_{92}Pu + ^{4}_{2}He \;\; T=24,360 \; years$$

This Pu is easily extracted from the Uranium because it is a different chemical *and so it can be chemically extracted (lets the electrons do the work). Enriching Uranium is much harder because the* ^{235}U *must be extracted from* ^{238}U *which is the same chemical and so chemical procedures cannot be utilized and centrifugal or mass spectrometry technology must be used.*

v) Do you think that nuclear power is a good thing for mankind?[4]

I think in principle that nuclear power is a very good thing. However, I think more money needs to be spent on research into rendering the nuclear waste harmless. I think the possibility of a reactor going critical is a risk, but if the reactors are placed in areas of very low population, then its concievable that the fatalities in such an event would be far fewer than those in a coal mine accident for example.

5. i) Describe the fusion reaction which takes place in the sun and the energy release of each step.[4]

The sun is in anequilibrium state where pressure and gravity balance. This happens when the interior becomes hot enough to ignite the Hydrogen burning cycle:

$$p + p \rightarrow ^2_1 H + e^+ + \nu_e + 0.42MeV$$
$$^2_1H + p \rightarrow ^3_2 He + \gamma + 5.49MeV$$
$$^3_2He + ^3_2 He \rightarrow ^4_2 He + p + p + 12.86MeV$$

ii) There are two effects in this reaction which cause the sun to burn very slowly. What are they and why are they of importance to us?[4]

This reaction occurs very *rarely. It not only must overcome the coulomb repulsion between the two protons but it must also rely on the weak interaction (which has very slow time scales because its so weak) for the beta decay. Thanks to this fact, the sun is still burning today. This reaction is so rare in fact that it has never been observed in a laboratory environment.*

iii) What are the major difficulties associated with reproducing fusion in a controlled environment?[4]

In order to produce the temperatures necessary to reach an energy of 60 keV (where the d-t cross section is a maximum) it is necessary to make a **plasma**. *This is a very hot gas which is essentially an ion gas because all the electrons have been boiled off their respective nuclei. These temperatures are very hard to sustain and would melt any container which tried to confine the gas. Therefore a pulsed device is used which heats the plasma for short bursts and at the same time the plasma is confined with a pulsed magnetic field (this, at least, cannot burn!)*