Solutions to Problem Sheet 4

3C24

March 21, 2000

1. i)

$$E_{nucleus} = Zm_p + Nm_n - B_{volume} - B_{surface} - B_{Coulomb} - B_{Symmetry} - B_{Pairing} nonumber \quad (1)$$
$$= Zm_p + Nm_n - u_v A + u_c Z^2 A^{-\frac{1}{3}} + u_s A^{\frac{2}{3}} + u_a T^2 A^{-1} \pm \frac{\delta}{2A}$$

Explain in detail each of the terms in this expression.[11]

First of all notice that the Coulomb term is $\frac{Z^2}{A^{1/3}}$ *and not* $\frac{Z(Z-1)}{A^{1/3}}$. *So don;t copy another formula out of the notes!!!*

u_v *The Volume Term: The strength of the binding of a nucleus is proportional to the number of nucleons inside. The strong force is charge independent and so this term does not discriminate between protons and neutrons.*

u_s *The Surface term: Nucleons at the surface of the nucleus will feel less attraction than those in the center. The magnitude of this effect is proportional to the surface area of the nucleus which is proportional to* $A^{2/3}$. *This term is charge independent: it does not discriminate between protons and neutrons.*

u_c *The Coulomb Term: The protons inside the nucleus repel each other because they are the same charge. The EM energy stored is proportional to* $\frac{e^2}{r}$ *and so the Coulomb term is proportional to* $\frac{Z(Z-1)}{A^{-1/3}}$. *This can be approximated to* $\frac{Z^2}{A^{-1/3}}$ *for large Z.*

u_a *The Asymmetry Term: The strong force is an exchange force. These are attractive if the wave funciton is symmetric under spacial exchange and repulsive if anti-symmetric. Anti-symmetric pairs do not contribute to the energy becasue they are the wrong sign for binding. The Pauli exclusion principle forces extra nuetrons into higher energy levels than they would be in if they were protons which will reduce the binding energy. The most stable nuclei will have fewest anti-symmetric pairs, i.e. N=Z. The term is proportional to* $(N-Z)^2$ *which gives a parabola.*

u_a *The Pairing Term: This is a purely empirical term which takes into account the fact that even-even nuclei are more stable than odd-odd and these in turn are more stable than odd A nuclei.*

ii) Obtain an expression for Z as a function of A for the stable isobars.[8]

For stable isobars

$$\left(\frac{\partial U}{\partial Z}\right) = m_p - m_n - 4u_a + 2\frac{u_c}{A^{1/3}}Z + 8\frac{u_a Z}{A}$$
$$= 0$$
$$Z = \frac{A}{2}\left[\frac{m_n - m_p + u_a}{u_c A^{2/3} + u_a}\right]$$

iii) Obtain an expression for the difference in energy of the parent nuclei and the decay products in alpha decay.[2]

$$\begin{aligned} \delta U &= U(Z,A) - U(Z-2,A-4) - U(2,4) \\ &= \frac{\partial U}{\partial Z}\Delta Z + \frac{\partial U}{\partial A}\Delta A - U(2,4) \end{aligned}$$

iv) Show it can be written in the form [8]

$$\frac{Z}{A^{\frac{1}{3}}}\left[\chi_1 - \chi_2\frac{Z}{A}\right] + \frac{\chi_3}{A^{\frac{1}{3}}} - \chi_4\left[1 - \frac{2Z}{A}\right]^2 - 28.1MeV \tag{2}$$

$$\begin{aligned} \left(\frac{\partial U}{\partial Z}\right) &= m_p - m_n - 4u_a + 2\frac{u_c}{A^{1/3}}Z + 8\frac{u_a Z}{A} \\ \left(\frac{\partial U}{\partial A}\right) &= m_n - u_v + \frac{2}{3}u_s A^{-1/3} - \frac{1}{3}u_c Z^2 A^{-4/3} + u_a - \frac{4Z^2 u_a}{A^2} \\ \Delta U &= 2\frac{\partial U}{\partial Z} + 4\frac{\partial U}{\partial A} - U(2,4) \end{aligned}$$

First, you need to know that the binding energy of an alpha particle, B(2,4)=28.3MeV.

$$\begin{aligned} &= 4u_c\frac{Z}{A^{1/3}} - 4u_v + \frac{8}{3}\frac{u_s}{A^{1/3}} - \frac{4}{3}\frac{u_c Z^2}{A^{4/3}} - 4u_a\left[\frac{A-2Z}{A}\right]^2 + 28.3MeV \\ &= \frac{Z}{A^{1/3}}\left[4u_c - \frac{4}{3}\frac{u_c Z}{A}\right] + \frac{8}{3}\frac{u_s}{A^{1/3}} - 4u_a\left[1 - 2\frac{Z}{A}\right]^2 + 28.3MeV - 4u_v \end{aligned}$$

At this point you need to know that

- *u_v =14.1MeV*

In order to turn $+28.3MeV - 4u_v$ *into -28.1MeV*

$$\frac{Z}{A^{1/3}}\left[\chi_1 - \chi_2\frac{Z}{A}\right] + \frac{\chi_3}{A^{\frac{1}{3}}} - \chi_4\left[1 - \frac{2Z}{A}\right]^2 - 28.1MeV \tag{3}$$

where

$$\begin{aligned} \chi_1 &= 4u_c \\ \chi_2 &= \frac{4}{3}u_c \\ \chi_3 &= \frac{8}{3}u_s \\ \chi_4 &= 4u_a \end{aligned}$$

2. i) Write down a general expression for the energy released in a nuclear decay

$${}^{A}_{Z}\mathrm{W} \rightarrow {}^{A-X}_{Z-Y}\mathrm{B} + {}^{X}_{Y}\mathrm{C}$$

in terms of parameters of the SEMF. [2]

$$\begin{aligned}\Delta B &= u_s\left[A^{2/3} - X^{2/3} - (A-X)^{2/3}\right] \\ &+ u_a\left[\frac{(A-2Z)^2}{A} - \frac{(X-2Y)^2}{X} - \frac{(A-X-2Z+2Y)^2}{(A-X)}\right] \\ &+ u_c\left[\frac{Z^2}{A^{1/3}} - \frac{Y^2}{X^{1/3}} - \frac{(Z-Y)^2}{(A-X)^{1/3}}\right]\end{aligned}$$

ii) Which parameter does not appear in the expression and why? [2]

u_v does not appear in the expression. The volume term is proportional to A and because the total number of nucleons is conserved in the decay the contribution to this term from the two daughters must be equal to that from the parents.

iii) Show that if the two decay fragments are of equal size, (i.e. X=A/2,Y=Z/2) that [4]

$$\begin{aligned}\frac{Z^2}{A} &> \frac{u_s(2-2^{2/3})}{u_c(2^{2/3}-1)} \quad (4)\\ &> 18 \quad (5)\end{aligned}$$

If X=A/2 and Y=A/2 then the asymmetry term u_a becomes:

$$\begin{aligned}B_a &= u_a\left[\frac{(A-2Z)^2}{A} - \frac{(A/2-Z)^2}{A/2} - \frac{(A/2-Z)^2}{A/2}\right] \\ &= 0\end{aligned}$$

so the expression for ΔB becomes

$$\begin{aligned}\Delta B &= u_s\left[A^{2/3} - 2(\frac{A}{2})^{2/3}\right] + u_c\left[\frac{Z^{2/3}}{A^{1/3}} - \frac{2(Z/2)^2}{(A/2)^{1/3}}\right] \\ &= u_sA^{2/3}\left[1-2^{1/3}\right] + \frac{u_cZ^2}{A^{1/3}}\left[1-(\frac{1}{2})^{2/3}\right] \\ &= u_sA^{2/3}\left[1-2(\frac{1}{2})^{2/3}\right] + \frac{u_cZ^2}{A^{1/3}}\left[1-2(\frac{1}{2})^{5/3}\right] > 0\end{aligned}$$

Using u_s=18.3MeV and u_c=0.7MeV:

$$\frac{Z^2}{A} > \frac{u_s[2(\frac{1}{2})^{2/3}-1]}{u_c[1-2(\frac{1}{2})^{5/3}]} > 18$$

iv) Use this expression to estimate the energy given out in fission of ^{235}U to two equal sized fragments. [2]

Substitute A=235, Z=92 into above expression. You get 174MeV

3. i) Explain how the strong and electromagnetic forces are involved in the existence of stable nuclei.[5]

There are two main effects which determine the stability of a particular nucleus:

* *The symmetry effect: the charge independence and the exchange character of nuclear forces, together with the Pauli exclusion principle, strongly depresses the energy of nuclei with equal or nearly equal numbers of protons and neutrons*
* *The charge effect: the effect of the Coulomb repulsion of the protons favours nuclei with fewer protons than neutrons. The neutron proton mass difference also enters the charge effect, but only as a small correction. The charge effect increases in importance with increasing Z*

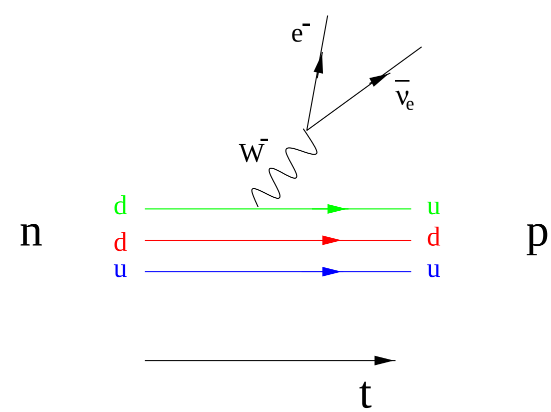
These are "non-bulk" effects: The volume term and the surface term are charge independent and are present irrespective of the composition of the nucleons (i.e. the relative numbers of protons and neutrons)

ii) Describe the three radioactive processes and explain the differences between them.[6]

Radioactivity is the name given to several distinctly different processes which we can now identify in terms of the fundamental forces which we have learned about. You may be familiar with the terms alpha, beta and gamma radiation, somewhat historically named.

Alpha radiation: (α) Alpha radiation is the spontaneous emission of a helium nucleus. This is a strong interaction process.

Beta radiation: (β) Beta radiation is just the weak decay of one of the quarks in a nucleon to an electron and an electron anti-neutrino. The electron is emitted (the beta ray) while the neutron(udd) becomes a proton(uud) via a W^- or under certain energetically favoured situations a proton becomes a neutron via a W^+. See Figure.



Feynman diagram of 'beta decay': a weak decay of a neutron to a proton, electron and anti-neutrino.

Gamma radiation: (γ) Gamma radiation is just another name for a particular energy of photon. These energies are in the MeV range and the photons come from the nucleons inside the nucleus rearranging themselves into a more energetically favourable configuration with the emission of the resultant energy given off as a photon. X rays on the other hand, come from electrons in the atoms rearranging themselves and dropping down across energy levels giving out photons. These photons are typically in the KeV range of energies.

iii) Explain why there are very few odd-odd even nuclei which are beta stable[4] *Experimentally, there is only one stable isobar for odd mass numbers but there are two and sometimes three for even mass numbers. These stable nuclei are always of the even-even variety with the odd-odd ones in between being beta-unstable in both directions. This is indicated in Figure 1(right) where U as a function of T is given for FIXED A. In this case, the difference between even-even and odd-odd nuclei is given purely*

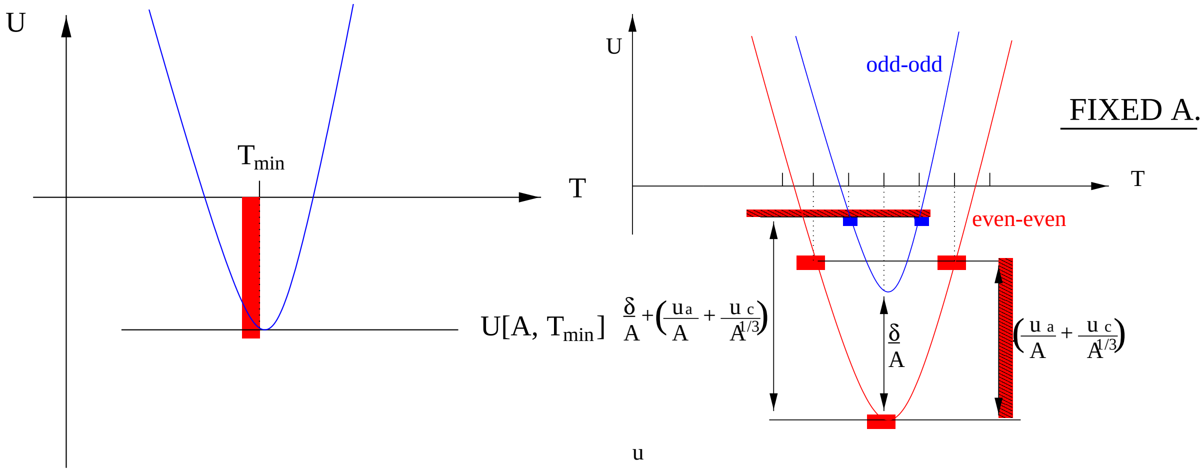


Figure 1: Left: U as a function of T from the SEMF. Right: U as a function of T for odd-odd and even-even nuclei for fixed A. The figure shows the case most favourable to the occurrance of three stable isobars (all of them even-even), with the spacing parameter δ chosen so large as to make the three even-even isobars stable.

by

$$\left(\frac{u_a}{A} + u_c A^{-1/3}\right) \tag{6}$$

The bulk terms being the same for a given A. The final term in the SEMF should be one depending on the type of the nucleus. The term added is

$$\pm\frac{\delta}{2A} \tag{7}$$

with the minus sign for even-even nuclei and the plus sign for odd-odd. This reflects what we learned earlier about the Deuteron: n-p pairs can be bound together because they can be in an S=1 state (their spins aligned parallel to each other) whereas n-n and p-p pairs to not add to the binding energy because the S=0 potential is not enough to bind the nucleon pairs. If there are an even number of protons and neutrons, this implies that there is maximal pairing between n and p nucleons whereas with odd-odd nuclei, it is at least guaranteed that there be an odd n and an odd p which are not paired. So even-even nuclei will have a larger binding energy than odd-odd nuclei given a fixed A. $\delta = 0$ for odd A. The proportionality to A^{-1} is assumed, since this is a symmetry effect. $\delta \approx 270 MeV$ for medium weight nuclei.

iv) Explain the requirements for X and Y isobars differing by one unit in Z being stable against beta decay and explain why this is a rare occurance. [4]

The difference in energy between the parent and daughter nuclei for beta decay can be written:

$$\Delta U_{Z\to Z+1,N\to N-1} = U(Z,N) - U(Z+1,N-1) \tag{8}$$
$$= B(Z+1,N-1) - B(Z,N) + (m_n - m_p) \tag{9}$$

We can look at three different general examples:

$$_ZX = {}_{Z+1}Y + e^- + \bar{\nu_e}; \Delta > 0.511\text{MeV} \tag{10}$$

$$
\begin{aligned}
{}_{Z+1}Y &= {}_{Z}X + e^{+} + \nu_e; \Delta < -0.511\text{MeV} && (11) \\
{}_{Z+1}Y + e^{-} &= {}_{Z}X + \nu_e; \Delta < 0.511 - \epsilon\text{MeV} && (12)
\end{aligned}
$$

where ϵ is the binding energy of the electron in the state from which it is captured. Any excess of energy appears as kinetic energy of the emitted electron and neutrino. Therefore we see that a nucleus is only *beta stable if either:*

$$
\begin{aligned}
\Delta U_{Z\to Z+1, N\to N-1} &< 0.511\text{MeV or} && (13) \\
\Delta U_{Z\to Z-1, N\to N+1} &< -0.511\text{MeV} && (14)
\end{aligned}
$$

this leads to the interesting result that two stable *isobars* must *differ by more than one unit in Z. X and Y isobars differing by one unit cannot be stable against beta decay* unless $\Delta U = 0.511 MeV$.