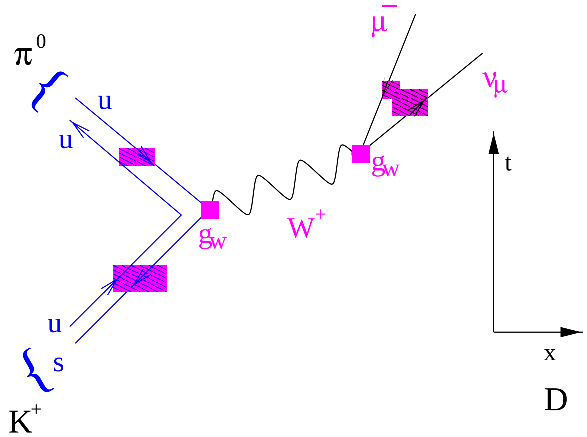
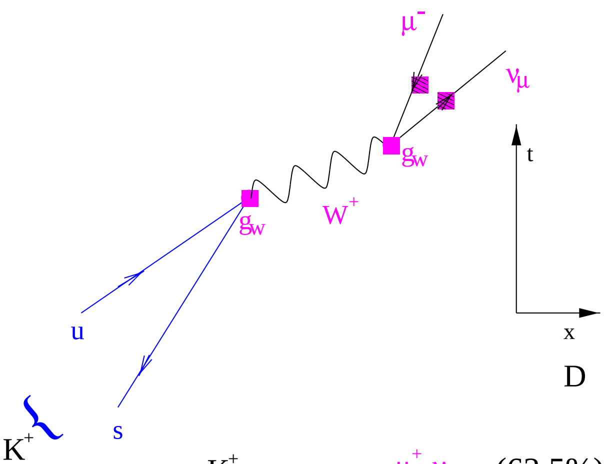
Solutions to Problem Sheet 2

3C24

February 23, 2001

1. i)





$K^+ \longrightarrow \pi^0\ \mu^+\ \nu_\mu$ (3.2%) $K^+ \longrightarrow \mu^+\ \nu_\mu$ (63.5%)

[4 + 4]

The annihilation decay, $K^- \to \mu\bar{\nu_\mu}$ dominates because there is more momentum space available to the (two) decay particles than in the case where there are three decay particle which have to share the available momentum. The 'available' momentum is called phase space. [2]

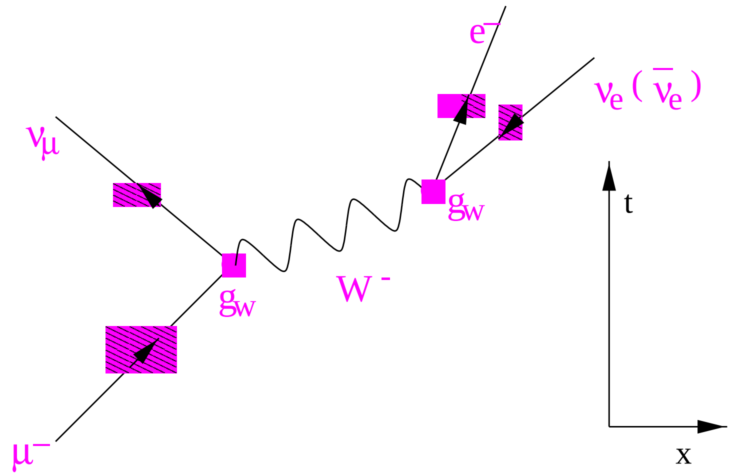
iii) *Of the two pion decays, $\pi \to e\bar{\nu_e}$ has more phase space available.* [1]

iv) *the decay $\pi \to \mu\bar{\nu_\mu}$ dominates because of parity violation in the Weak Interaction. There are only left-handed neutrinos and right-handed anti-neutrinos and so in order to conserve angular momentum in the decay, the muon must have positive helicity. However, because this is a Weak Interaction the muon must be produced in the left-handed Chiral State which is a combination of a positive and negative helicity state.*

$$\begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi'_- \\ \chi'_+ \end{pmatrix} \qquad (1)$$

The relative amount of positive and negative helicity components is determined by how relativistic the muon is, or alternatively, how massive the particle is. For a very relativistic muon, $\chi_{LH} \approx \chi'_{-ve}$ and of course for neutrinos, $\chi_{LH} \equiv \chi'_{-ve}$. For the charged pion decay, the momentum of the muon or electron is about 30MeV. The electron, being very light, is very relativistic at this momentum whereas the muon is slower. Therefore, the Weak decay favours the muon decay because the fraction of the left-handed Chiral state in the positive helicity state is higher.8] [[]

[2]

i) 

[[]
4]

ii) *Either this one:*

$$\Gamma \quad = \quad \int |\mathcal{M}|^2 \delta^4 (p_{\mu^-} - p_{\nu_\mu} - p_{e^-} - p_{\bar{\nu_e}}) \rho_f d\Omega \qquad (2)$$

where

- γ = *width (in units of energy) of the state* [1]
- $|M|$ = *Invariant Amplitude* [1]
- ρ_f = *density of states: total number of combinations for the final energy* [1]
- $d\omega$ = *incremental solid angle* [1]
- δ^4 = *a four dimensional delta function which* ensures *energy and momentum conservation* [1]

or

$$\chi(E) \quad = \quad \frac{K}{(M - E) - i\Gamma/2} \qquad (3)$$

where

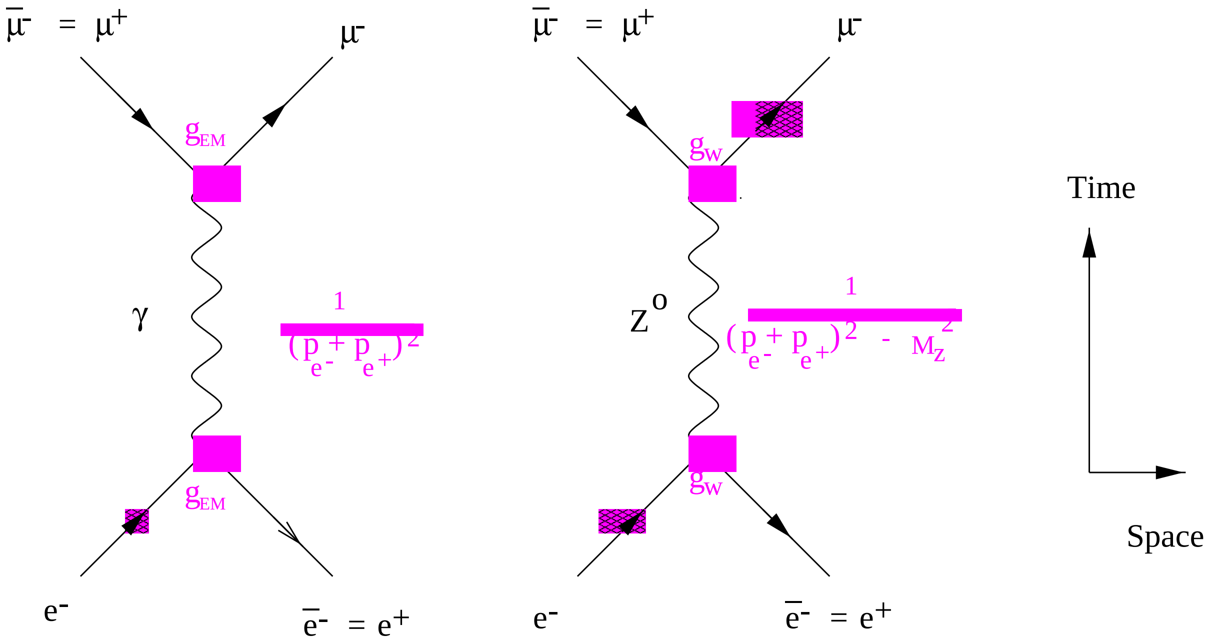
- γ = *width (in units of energy) of the state* [1]
- $\chi(E)$ = *energy dependent Invariant Amplitude* [1]
- M = *mass of the resonance* [1]
- E = *total energy* [1]
- K = *a proportionality constant which includes the time independent part of the wave function* [1]

iii) If the muon lifetime is 2.2μsec, what is the width?

$$\gamma = \frac{1}{2.2 \cdot 10^{-6}} \cdot \frac{1}{1.519 \cdot 10^{-19}} \qquad (4)$$

for the first part you get one mark, for the second you get one mark [2]

iv) Which is easier to measure? *The lifetime is easier to measure in this case. The muon can travel a long distance in $2.2\mu s$ which can be measured and used to deduce the lifetime. The width, on the other hand, is very narrow and it is not possible to produce a muon as a resonance with a convertional accelerator.*



3. i) Draw the Feynman Diagrams for e^+e^- annihilation.[2]

ii) Explain why e^+e^- annihilation is such a good microscope on new particles. *Two points:*

- *Its a very economical use of the center of mass energy(the energy available to make new particles) compared to a fixed target interaction. For e^+e^- annihilation, the available energy for a new particle is $m^* = E_{beam}$ wheras for fixed target the available energy $m^* = \sqrt{2m_{target}E_{beam}}$* [2]
- *Because the interaction can be mediated via a virtual photon or a virtual Z, all the fermions that we know about have a chance to be produced (as long as there is enough energy). The Z even couples to neutrinos whereas the photon only couples to charged particles* [2]

iii) At 9GeV, the weak diagram can be ignored. What should be the value of

$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \quad (5)$$

$$= 3\Sigma e_q^2 \quad (6)$$

$$(7)$$

At 9 GeV, no top or bottom quarks can be produced. Therefore

$$\sum e_q^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 \quad (8)$$

$$R = 3\frac{10}{9} \quad (9)$$

[2]

iv) Why does R tell us about the number of colours? *In the absence of any extra quantum number, the value of R should be $\frac{10}{9}$. The fact that it is about three times that tells us that there are three new degrees of freedom which the quarks can have which correspond to the three allowed values of a new quantum number called colour.* [2]

fermion	d	u	s	c	b	e	μ	τ	ν_e	ν_μ	ν_τ
g_{EM}	$\frac{-1}{3}$	$\frac{2}{3}$	$\frac{-1}{3}$	$\frac{2}{3}$	$\frac{-1}{3}$	-1	-1	-1	0	0	0

v) Write down Q_{EM} for all the fermions[4]

vi) Show why R$\propto$ e_q^2 *R is a ratio of cross-sections. The cross section for an Electromagnetic interaction is proportional to the square of the Invariant Amplitude so the ratio R will be gievn by the ratio of thesquares of the two Invariant Amplitudes, in once case* $(Q_e Q' q g_{EM}^2)^2$ *and in the other case* $(Q_e Q' \mu g_{EM}^2)^2$. *In the ratio everything cancels except* $\frac{(Q_q')^2}{(Q'\mu)^2}.(Q'\mu)^2 = 1$ *so* $R \propto Q_q^2$

3. i) In e^+e^- annihilation near a resonance, the expression for the invariant amplitude has to be derived taking into account the time dependence of the particle wave function. Why? *In order to calculate the Invariant Amplitude for resonant production, attention has to be paid to the time dependent part of the wave function because the lifetime of the new resonant state is not necessarily short compared to the time evolution of the wavefunction and so it cannot be ignored* [2]

ii) Starting from

$$\phi(t) = \psi(0)e^{-E_R t}e^{-t/2\tau} \quad (10)$$
$$= \psi(0)e^{-t[iE_R+\Gamma/2]} \quad (11)$$

show that the Invariant Amplitude is proportional to

$$\frac{1}{p^2 - m^2 - \frac{i\Gamma}{2}} \quad (12)$$

at, or very close to, the resonance.[4]

$$\begin{aligned} |\psi(t)|^2 &= |\psi(0)|^2 e^{-\Gamma t} \text{ then} \\ \psi(t) &\propto e^{-iE_R t}e^{-\frac{\Gamma t}{2}} \\ \phi(t) &= \psi(0)e^{-E_R t}e^{-t/2\tau} \\ &= \psi(0)e^{-t[iE_R+\Gamma/2]} \end{aligned}$$

The Fourier transform is

$$\begin{aligned} \chi(E) &= \int_0^\infty \phi(t)e^{iEt}dt \\ &= \psi(0)\int_0^\infty e^{-t[\Gamma/2+iE_r-iE]}dt \\ &= \frac{K}{(M-E)-i\Gamma/2} \end{aligned}$$

Get 1 mark for this so far: This is now the energy dependent Invariant Amplitude. However, it doesn't really look all that much like what we had before! Actually, it is almost the same. Start with the form of our time independent Invariant Amplitude:

$$\begin{aligned} \frac{1}{q^2 - M^2} &= \frac{1}{E^2 - m^2} \quad (\mathbf{p} = 0 \text{ for e}^+\text{e}^- \text{ annihilation}) \\ &= \frac{1}{(E-M)(E+M)} \text{ at resonance E} \approx \text{M so} \\ &= \frac{1}{-2M(M-E)} \end{aligned}$$

So, if $q^2 - M^2 = -2M(M-E)$ *then*

$$\frac{K}{(M-E) - i\Gamma/2} \propto \frac{1}{q^2 - M^2 - i\Gamma/2} \tag{13}$$

You get three marks for doing this correctly

4. i) Why are neutrinos different from all other fermions?[2]

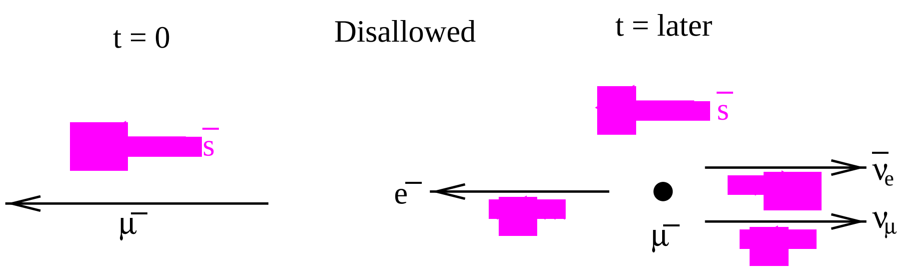
   two reasons:

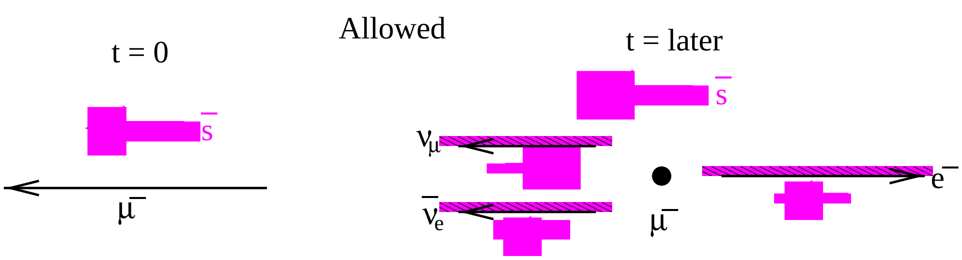
   - $*$ *neutrinos only interact weakly unlike the other fermions* [1]
   - $*$ *because they are massless, they only have one helicity: negative* [1]

   ii) Why does parity violation in the weak interaction mean that neutrinos must be massless? (At least in the Standard Model framework)[4]

   Because the weak force is only seen by left-handed (chiral state) particles it is called parity violating. The handedness of a particle comes from the value of its chirality. Neutrinos are only produced in the weak interaction and therefore if a right handed neutrino were to exist, it would have no allowed interaction at all. If it had no allowed interaction at all, how could it be produced? The only way to get out of the apparent puzzle is to insist that neutrinos be massless. If that is the case, a left handed neutrino (with negaitive helicity) cannot be boosted into a frame where it has positive helicity (if both helicity states existed, it would imply the existance of right-handed neutrinos) because special relativity insists that a particle travelling at $v = c$ *will travel in every frame at* $v = c$*. The fact that no frame exists for a neutrino to have positive helicity leaves no way in which a right handed particle can be produced from a left handed particle.* [4]

   iii) *for the most energetic electrons, the electron will recoil against the two neutrinos, receiveing about half the energy of the mass of the muon*





   iv) *there are two separate parts to this answer. The first is that when the muon is produced via the weak interaction, it will be left handed. The muon decays to two* *neutrinos and an electron. The decay can have two distinct orientations with respect to the muon spin direction. These are shown in the figure (over the page) . The neutrino MUST be left-handed and the antineutrino MUST be right-handed. The electron then has two possibilities. In the top diagram, it is right-handed and in the bottom diagram it is left-handed. Because the muon decay is a weak interaction, it will produce a left-handed electron also. Therefore the top diagram is disfavoured.* [2] [2]

v) this equation should say : how do you produce a negative helicity muon at rest? *Even though helicity is not conserved for non-relativistic particles (such as the muon will be when it comes to rest) if the muon is produced in a weak interaction such as pion decay and it slows down in the absence of a magnetic field (which can cause the direction of spin to change) its initial spin direction will be preserved. This is how to produce a negative helicity muon at rest. This spin direction sets up the coordinate system of the problem.* [2]

Please go over your answers carefully. If you think you should have received more marks for any part of a question, please bring your problem sheet to the next lecture and I will take a look at it. J.T.