Solutions to Problem Sheet 1

3C24

February 23, 2001

1. Calculate the range of the forces transmitted by gauge bosons of mass:[3]

   (i) m = 100 GeV

   (ii) m = 100 MeV

   (iii) m = 5 Gev (hypothetical)

   Calculate the equivalent energy of the distance:[3]

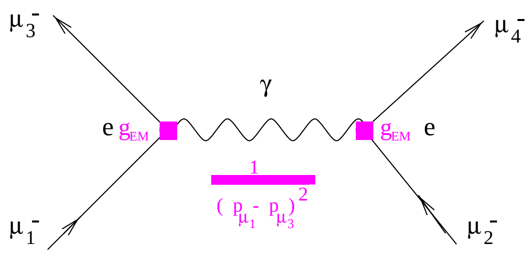
   (i) 10^{-15}m (1fm)

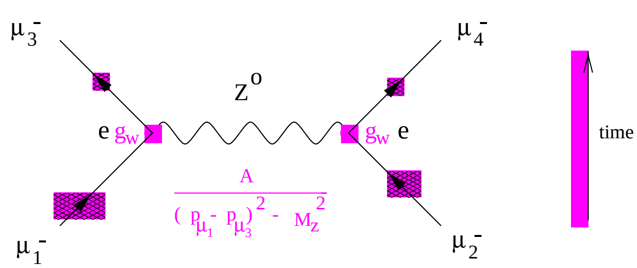
   (ii) 10^{-18}m

   (iii) 10^{-9}m (1nm)

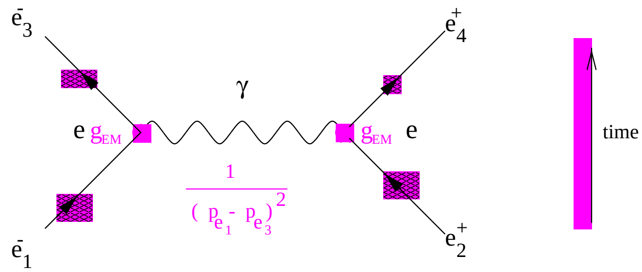
2. Draw Feynman Diagrams for the following processes and show that the quantum numbers Baryon number, Lepton (flavour) number, electric charge and colour are conserved at the vertices and identify which Gauge Boson is being exchanged:

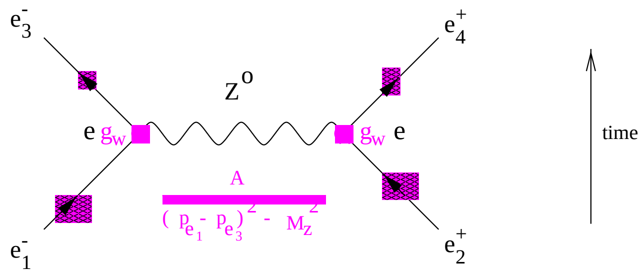
   (i) $\mu^-\mu^- \to \mu^-\mu^-$[6]



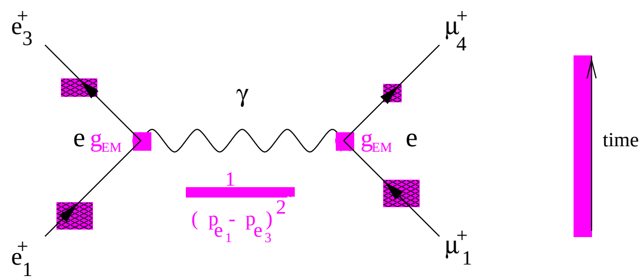


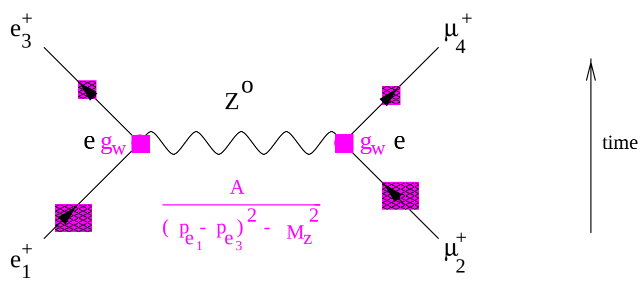
   (ii) $e^-e^+ \to e^-e^+$[6]



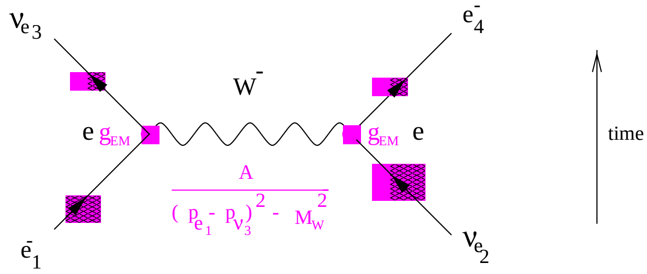


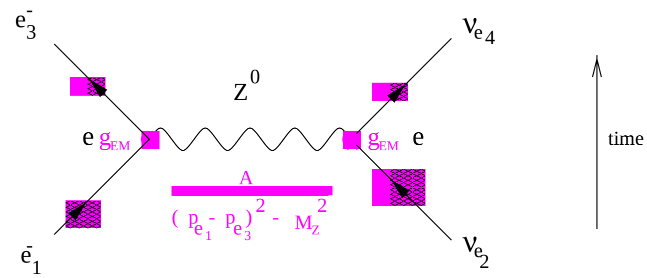
(iii) $e^+\mu^+ \to e^+\mu^+[6]$





(iv) $e^-\nu_e \to \nu_e e^-[6]$





3. (i) Derive the expression for the Invariant Amplitude from the premise that Perturbation Theory is valid.[5]

$$\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1 \tag{1}$$

$$V_{21} = g_2 \int e^{-i\mathbf{q}\cdot\mathbf{r}} V(r) d\mathbf{r} \tag{2}$$

$$\mathbf{q}\cdot\mathbf{r} = qr\cos\theta \tag{3}$$

$$d\mathbf{r} = r^2 d\phi \sin\theta d\theta dr \tag{4}$$

For **r**: -$\infty \to \infty$ For r: $0 \to \infty$

$$V(r) = \frac{g_2}{4\pi r} e^{-mr} \tag{5}$$

$$V_{21} = \frac{g_1 g_2}{4\pi} \int e^{-iqr\cos\theta} \frac{e^{-mr}}{r} r^2 d\phi d\cos\theta dr \tag{6}$$

$$= 2\pi \frac{g_1 g_2}{4\pi} \int_0^\infty r^2 f(r) dr \int_{-1}^{1} e^{-iqr\cos\theta} d\cos\theta \tag{7}$$

Change of variable

$$\begin{aligned} \lambda &= -iqr\cos\theta : & e^\lambda &= e^{-iqr\cos\theta} \\ d\lambda &= -iqr\ d\cos\theta : & d\cos\theta &= \tfrac{d\lambda}{iqr} \\ \cos\theta &= \pm 1 : & \lambda &= \mp iqr \end{aligned}$$

$$\int_{-iqr}^{iqr} \frac{d\lambda}{iqr} e^\lambda = \left[\frac{e^\lambda}{iqr}\right]_{iqr}^{-iqr}$$

$$V_{21} = \int_0^\infty e^{-mr} r dr \frac{1}{iqr} (e^{-iqr} - e^{iqr})$$

$$= \frac{q_1 g_2}{2iq} \int_0^\infty e^{-mr} (e^{-iqr} - e^{iqr}) dr$$

$$= \frac{q_1 g_2}{2iq} \left[\frac{e^{-r(m+iq)}}{-(m+iq)} - \frac{e^{-r(m\frac{q_1 g_2}{m^2+\mathbf{q}^2} - iq)}}{-(m-iq)} \right]_0^\infty$$

$$
\begin{aligned}
&= \frac{q_1 g_2}{2iq}\left[\frac{-(m-iq)e^{-mr}e^{-iqr}+(m+iq)e^{-mr}e^{iqr}}{m^2+\mathbf{q}^2}\right]_0^\infty \\
&= -\frac{q_1 g_2}{m^2+\mathbf{q}^2} \qquad (8)
\end{aligned}
$$

(ii) At a distance scale of 10^{-18}m, compare the relative Invariant Amplitudes for Strong (colour), Weak and Electromagnetic Forces. (Ignore the energy dependence of the coupling strength).[2+3]

Distance of $10^{-18}m \rightarrow E = 100GeV$.

- $IA_{strong} = (0.14/10^4)/(0.14/10^4) = 1.$
- $IA_{em} = (0.008/10^4)/(0.14/10^4) = 0.06$
- $IA_{weak} = (0.034/2 \text{ X } 10^4)/(0.14/10^4) = 0.12$

Look!The weak interaction is NOT the weakest at this energy scale!

(iii) At a momentum transfer of 10GeV, compare the relative Invariant Amplitudes for Strong (pion), Weak and Electromagnetic Forces. (Ignore the energy dependence of the coupling strength).[3]

- $IA_{strong} = (0.14/10^2)/(0.14/10^2) = 1.$
- $IA_{em} = (0.008/10^2)/(0.14/10^2) = 0.06$
- $IA_{weak} = (0.034/(10^2 + 10^4))/(0.14/10^2) = 0.002$

Weak force is the weakest here because of supression of the massive gauge boson

4. (i) Write down the expression relating the Invariant Amplitude to the cross section and explain why we are interested in this relation.[1+4]

The cross section is given by

$$\sigma = \frac{|\mathcal{M}|^2\delta^4(p_{e_i}+p_{\mu_i}-p_{e_f}-p_{\mu_f})(2\pi)^4\rho_{E_i}}{\phi} \qquad (9)$$

$$\sigma = \frac{|\mathcal{M}|^2\delta^4(p_{e_i}+p_{\mu_i}-p_{e_f}-p_{\mu_f})\rho_f}{\phi} \qquad (10)$$

and has units of area. The delta function just represents momentum and energy conservation. ρ_f is a phase space factor which absorbs the density of states and the numerical constants. We are intersted in this relation becasue it relates a measurable quantity to the gauge theory

(ii) Explain the concept of **density of states**.[3]

The final state which is measured is only one of a set of allowed final states. For example, for a total energy of 2GeV, the final state particles could have 1Gev each, 0.5Gev and 1.5Gev, 0.2GeV and 1.8GeV etc etc. The number of states in this energy interval is proportional to the number of different allowed decays. The number of different allowed decays depends on the energy of the incoming particles. So if there are 4 different final state channels allowed, the density of states will be 4 times $\rho(E_f)$ if there were only one allowed channel.