UNIVERSITY COLLEGE LONDON
DEPARTMENT OF PHYSICS AND ASTRONOMY

2B21 MATHEMATICAL METHODS IN PHYSICS AND ASTRONOMY

Problem Sheet M2 (2003–2004)

Solutions to be handed in on Tuesday 21 October 2003

1. The equations

$$\begin{aligned} x_2 &= \tfrac{1}{\sqrt{2}}(x_1 - z_1) & x_3 &= \tfrac{1}{\sqrt{2}}(y_2 - z_2)\,, \\ y_2 &= \tfrac{1}{2}(x_1 + \sqrt{2}y_1 + z_1) & y_3 &= -\tfrac{1}{2}(\sqrt{2}x_2 + y_2 + z_2)\,, \\ z_2 &= \tfrac{1}{2}(x_1 - \sqrt{2}y_1 + z_1) & z_3 &= \tfrac{1}{2}(-\sqrt{2}x_2 + y_2 + z_2) \end{aligned}$$

represent rotations in three dimensions. Use matrix techniques to express the components of $\underline{r}_3$ in terms of those of $\underline{r}_1$.

What does the resultant single transformation represent geometrically? **[7 marks]**

2. Use Cramer's rule to solve the simultaneous equations

$$\begin{aligned} 3x_1 - 2x_2 - x_3 &= 4\,, \\ 2x_1 + x_2 + 2x_3 &= 10\,, \\ x_1 + 3x_2 - 4x_3 &= 5 \end{aligned}$$

to find the values of x_1, x_2, and x_3. **[8 marks]**

3. The matrices $\underline{A}$, $\underline{B}$, and $\underline{D}$ are related by $\underline{D} = \underline{A}\,\underline{B}$. Given that

$$\underline{A} = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 3 & 2 \end{pmatrix} \quad \text{and} \quad \underline{D} = \begin{pmatrix} 10 & -6 & 6 \\ 9 & -6 & 5 \\ 15 & -10 & 11 \end{pmatrix}\,,$$

evaluate $\underline{A}^{-1}$. **[7 marks]**

Hence derive the value of $\underline{B}$. **[3 marks]**