

PHAS1245: Mathematical Methods I - Problem Sheet 8

(Solutions to be handed in at the lecture on Tuesday 4th December 2007)

Staple your answer sheets together and put **your name** and your **tutor's name** on your script (or Dr. Konstantinidis, if you have no tutor in the P&A department).

1. If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ show that $|z_1 z_2| = |z_1| |z_2|$.
2. By considering the real and imaginary parts of the product $e^{i\theta} e^{i\phi}$, derive the standard expressions for $\cos(\theta + \phi)$ and $\sin(\theta + \phi)$.
3. (a) Evaluate $\operatorname{Re}(e^{2iz})$, where $z = x + iy$.
(b) Use the Argand diagram to show that $(-1 + i\sqrt{3}) = 2e^{2\pi i/3}$. Then determine $(-1 + i\sqrt{3})^{1/2}$.
(c) Evaluate $\left(\frac{\sqrt{2}}{i-1}\right)^6$.

4. Evaluate the integral

$$\int_0^\pi \sin 3x \cos 4x \, dx.$$

5. Find all five roots of $\sqrt[5]{-1 - i}$.