PHAS1245: Mathematical Methods I - Problem Sheet 5

(Solutions to be handed in at the lecture on Tuesday 13th November 2006)

Staple your answer sheets together and put **your name** and your **tutor's name** on your script (or Dr. Konstantinidis, if you have no tutor in the P&A department).

1. If $\vec{A} = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$ and $\vec{B} = 2\hat{\imath} + 7\hat{\jmath} + \hat{k}$, find **[6]**

$$\vec{A} + \vec{B} \quad , \quad \vec{A} - \vec{B} \quad , \quad |\vec{A}| \quad , \quad |\vec{B}| \quad , \quad \vec{A} \cdot \vec{B}$$

   and the cosine of the angle between the two vectors.

2. Find the unit vector perpendicular to $(\hat{\imath} + \hat{\jmath} - \hat{k})$ and $(2\hat{\imath} - \hat{\jmath} + 3\hat{k})$. **[2]**

3. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b} = -\hat{\imath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$, obtain **[4]**

$$\vec{a} \cdot \vec{b} \quad , \quad \vec{a} \times \vec{b} \quad , \quad \vec{a} \cdot (\vec{b} \times \vec{c}) \quad \text{and} \quad \vec{a} \times (\vec{b} \times \vec{c}).$$

4. Let $\vec{A}$ be an arbitrary vector and $\hat{n}$ a unit vector pointing in an arbitrary direction. Show that $\vec{A}$ may be expressed as **[3]**

$$\vec{A} = (\vec{A} \cdot \hat{n})\hat{n} + (\hat{n} \times \vec{A}) \times \hat{n} \,.$$

5. Find the angle between the position vectors to the points $(3, -4, 0)$ and $(-2, 1, 0)$ and find the direction cosines (i.e. the cosines of the angles with respect to the $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ unit vectors) of a vector perpendicular to both position vectors. **[6]**

6. Use the vector product to determine the direction of the line of intersection of the two planes $x + 2y + 3z = 0$ and $3x + 2y + z = 0$. Find the direction cosines of the line of intersection. Repeat the above by determining two points common to both planes. **[5]**