# PHAS1245: Mathematical Methods I - Problem Sheet 4

(Solutions to be handed in at the lecture on Tuesday 30th October 2007)

**Staple** your answer sheets together and put **your name** and your **tutor's name** on your script (or Dr. Konstantinidis, if you have no tutor in the P&A department).

1. Starting from **[3]**

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

show that

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2t}{1+t^2} \qquad (t = \tan\frac{x}{2})\,.$$

Hence show that **[5]**

$$\int \frac{1}{\sin x}\,dx = \ln\ (\tan\frac{x}{2})\,.$$

2. Given that **[4]**

$$\int_{-\infty}^{\infty} \mathrm{e}^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}\,,$$

show that

$$\int_0^{\infty} x^2 \mathrm{e}^{-\alpha x^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}\,.$$

3. The distribution of the speed $v$ of molecules, mass $m$, in a gas in thermal equilibrium at temperature $T$ is given by **[4]**

$$P(v)dv = Av^2 \mathrm{e}^{-\frac{1}{2}\frac{mv^2}{kT}}\,dv$$

where $k$ is the Boltzmann constant and $A$ is a normalizing constant. Determine $A$ such that $\int_0^\infty P(v)dv = 1$ .

4. From the definitions of $\cosh z$ and $\sinh z$ show that **[2]**

$$\cosh z + \sinh z = \mathrm{e}^z \qquad\qquad \cosh^2 z - \sinh^2 z = 1\,.$$

5. Evaluate **[5]**

$$\int \frac{1}{\sqrt{x^2+6x+1}}\,dx$$

(hint: you will eventually need to use hyperbolic functions).