PHAS1245: Mathematical Methods I - Problem Class 5

Week starting Monday 3rd December

NOTE: the topics addressed in this paper are partial differentiation (questions 1, 3, 5) and complex numbers (questions 2, 4, 6).

1. Find the first partial derivatives of

$$\theta(x, y) = \arctan\left(\frac{y}{x}\right) .$$

2. If $z = x + \imath y$, find the equation in terms of x and y of the sets of points in the complex plane that satisfy $\mathrm{Re}(z^2) = \mathrm{Im}(z^2)$.

   [Answer: $x^2 - 2xy - y^2 = 0$]

3. Find the stationary points of the function

$$f(x, y) = x^3 - y^3 - 2xy + 2$$

   and determine their nature.

   [Answer: the points are (0,0) and $(\frac{2}{3}, \frac{2}{3})$. The first is undertermined the second is a minimum.]

4. By writing $\pi/12 = \pi/3 - \pi/4$ and considering $\mathrm{e}^{\imath\pi/12}$, evaluate $\cot(\pi/12)$ without using a calculator.

   [Answer: $2 + \sqrt{3}$]

5. The function $f(x, y)$ satisfies the equation

$$y\frac{\partial f}{\partial x} + x\frac{\partial f}{\partial y} = 0 .$$

   By changing to new variables $u = x^2 - y^2$ and $v = 2xy$, show that f is, in fact, a function of $x^2 - y^2$ only.

6. Find the roots of

$$\sqrt[4]{\imath 8\sqrt{3} - 8}$$

   and sketch them in the Argand diagram.

   [Answer: $\mathrm{e}^{\imath\pi/6}$, $\mathrm{e}^{\imath 4\pi/6}$, $\mathrm{e}^{\imath 7\pi/6}$, $\mathrm{e}^{\imath 10\pi/6}$]