PHAS1245: Mathematical Methods I - Problem Class 2

Week starting Monday 29th October

1. Determine the following integrals:

$$\text{(a)} \int x(3x^2 - 2)\, dx \qquad \text{(b)} \int \frac{1}{x \ln^2 x}\, dx \qquad \text{(c)} \int_{-\pi/2}^{\pi/2} \cos^3\theta\, d\theta$$

$$\text{(d)} \int e^{\cos x} \sin x\, dx \qquad \text{(e)} \int \frac{1}{t^2} \sin\left(\frac{1}{t}\right) dt \qquad \text{(f)} \int \frac{\sin 2x}{1+\cos^2 x}\, dx\,.$$

2. Use integration by parts (or another method of your choice) to evaluate the following integrals:

$$\int \frac{\ln(a^2+x^2)}{x^2}\, dx \qquad \int x^3(1-x^2)^3\, dx \qquad \int x^r \ln x\, dx\,, \quad r \neq -1\,.$$

3. Evaluate the integral

$$I = \int \frac{1}{ax^2+bx+c}\, dx\,,$$

with $a \neq 0$, distinguishing between the cases (i) $b^2 > 4ac$, (ii) $b^2 = 4ac$ and (iii) $b^2 < 4ac$.

4. Show that

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1})\,.$$

5. Evaluate the integral

$$I = \int \sqrt{x^2+4x+13}\, dx\,.$$

6. A water container used as a water clock has depth 0.5 m and its shape is given by $r(h) = 0.39h^{1/4}$, where $r(h)$ is its radius at height h from its bottom. At the bottom there is an outlet and the size of its hole is such as to drain the water at a rate given by

$$\frac{dV}{dt} = -0.003\sqrt{h}$$

cubic metres per hour, where V is the volume of water remaining. (a) Determine the volume of the container (hint: this is a surface/volume of revolution.) (b) Show that the water level falls at a uniform rate and find how long it runs. (Consider the change δh in level which occurs in a short time δt.)