# PHAS1245 - Problem Class 6 - Solutions

1.

2. Use L'Hopital's rule in both cases as the limit goes to 0/0.

$$\lim_{x\to 0}\frac{x\mathrm{e}^{-x}}{1-\mathrm{e}^{-x}} = \lim_{x\to 0}\frac{\frac{d}{dx}(x\mathrm{e}^{-x})}{\frac{d}{dx}(1-\mathrm{e}^{-x})} = \lim_{x\to 0}\frac{\mathrm{e}^{-x}-x\mathrm{e}^{-x}}{\mathrm{e}^{-x}} = \lim_{x\to 0}(1-x) = 0$$

Similarly (only we have to go to the second derivatives in this case as the limit of the first derivatives is again 0/0):

$$\lim_{x\to 0}\frac{\tan x - x}{\cos x - 1} = \lim_{x\to 0}\frac{\frac{1}{\cos^2 x}-1}{-\sin x} = \lim_{x\to 0}\frac{-2\frac{1}{\cos^3 x}(-\sin x)}{-\cos x} = \lim_{x\to 0}\left(-\frac{2\sin x}{\cos^4 x}\right) = 0\,.$$

3.

4. The sum is a geometric series with ratio $R\mathrm{e}^{\imath\phi}$. Hence, we can use the expression derived in the lectures for a finite geometric series and take the limit for $N\to\infty$:

$$\sum_{n=0}^{\infty} R^n \mathrm{e}^{\imath n\phi} = \lim_{N\to\infty}\frac{(1-(R\mathrm{e}^{\imath\phi})^N)}{1-R\mathrm{e}^{\imath\phi}} = \frac{1}{1-R\mathrm{e}^{\imath\phi}} \Rightarrow B = A(1-R)\frac{1}{1-R\mathrm{e}^{\imath\phi}}$$

(since $R<1$ the series converges)

$$\Rightarrow |B|^2 = A^2(1-R)^2\frac{1}{(1-R\mathrm{e}^{\imath\phi})(1-R\mathrm{e}^{-\imath\phi})} = A^2(1-R)^2\frac{1}{1+R^2-R(\mathrm{e}^{\imath\phi}+\mathrm{e}^{-\imath\phi})}$$

5.