

Relativity & Gravitation

SPA7019

by

Dr Timothy Clifton

Course outline

Einstein's theory of relativity is one of the pillars of modern physics, and is currently enjoying a renaissance due to recent progress in cosmology and gravitational wave detection. This course is aimed at providing sufficient tools to understand the deep physics that underpins these advances, and to provide the foundational mathematics and physics required for more advanced study. This will begin with an introduction to differential geometry, before moving on to Einstein's gravitational field equations and their solutions. It will include the study of black hole physics and gravitational wave emission.

Syllabus

- Introduction to differential geometry, including an explanation of how physics should be understood in curved spaces.
- Presentation of Einstein's theory of general relativity, including some exact solutions to the field equations of the theory (i.e. black holes)
- Perturbative relativistic gravity, for use in the Solar System and for calculating the gravitational wave signals from inspiralling binaries (as detected by LIGO).

Prerequisites

Knowledge of Newtonian mechanics and special relativity. Basic knowledge of general relativity would also be beneficial.

Books

General Relativity: An Introduction for Physicists by M. P. Hobson, G. Efstathiou and A. N. Lasenby, Cambridge University Press, 2006.

Assessment

Written examination of 2½ hours contributing 90%, and problem sheets contributing 10%.

Assumed knowledge

I will assume that you know, and are comfortable with, the concepts below. If this is not the case, then come and see me immediately.

- The use of indices to denote sets of objects (such as components of vectors)
- The summation convention (i.e. $x_i^i = \sum_i x_i^i$)
- That units can be chosen such that $c = 1$ (this will be used throughout)
- The basics of special relativity, including the use of Minkowski space and position and momentum 4-vectors
- The Kronecker delta and the Dirac delta (not the same thing!)
- The basic rules of calculus (product rule, chain rule, etc).

Contents

1. Geometry & space-time
 - 1.1 Manifolds
 - 1.2 Coordinates
 - 1.3 Curves and surfaces
 - 1.4 Tangent vectors
 - 1.5 Tangent spaces
 - 1.6 Vectors and vector fields
 - 1.7 Basis vectors
 - 1.8 The metric
 - 1.9 Dual basis
 - 1.10 Coordinate transformations
 - 1.11 Outer product
 - 1.12 Tensors
 - 1.13 Tensor operations
 - 1.14 Connection
 - 1.15 The Levi-Civita connection
 - 1.16 Covariant derivative
 - 1.17 Intrinsic derivative
 - 1.18 Parallel transport
2. Einstein's theory & black holes
 - 2.1 Observables
 - 2.2 Observers & frames
 - 2.3 Frame transformations
 - 2.4 Lorentz transformations
 - 2.5 Light rays
 - 2.6 Frequency of light
 - 2.7 Stress-energy tensor
 - 2.8 Stress-energy conservation
 - 2.9 Inertial frames
 - 2.10 Riemann curvature tensor
 - 2.11 Ricci and Weyl tensors
 - 2.12 Einstein's equations
 - 2.13 Empty space I: Minkowski
 - 2.14 Empty space II: Milne
 - 2.15 Empty space III: Plane waves
 - 2.16 Empty space IV: de Sitter space
 - 2.17 One-body solution I: Schwarzschild
 - 2.18 One-body solution II: Kerr
 - 2.19 One-body solution III: General case
 - 2.20 Multi-body solutions

- 3. Gravitational waves & perturbation theory
 - 3.1 Perturbation theory
 - 3.2 Gauge transformations
 - 3.3 The Lorenz gauge
 - 3.4 Linearised solutions in vacuum
 - 3.5 Transverse-traceless gauge
 - 3.6 The effect of gravitational waves
 - 3.7 Linearised solutions with sources
 - 3.8 The quadrupole formula
 - 3.9 Static sources and Newtonian limit
 - 3.10 Waves from binary systems
 - 3.11 Gravity of gravitational waves
 - 3.12 Energy radiated from a binary
 - 3.13 The Hulse-Taylor binary
 - 3.14 The LIGO detections