UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Physics 1B24: Waves, Optics and Acoustics

COURSE CODE	:	**PHYS1B24**
UNIT VALUE	:	**0.50**
DATE	:	**14-MAY-02**
TIME	:	**10.00**
TIME ALLOWED	:	**2 hours 30 minutes**

02-C1068-3-170

TURN OVER

Answer SIX questions from section A and THREE questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

SECTION A **[Part marks]**

1. Write down the partial differential equation governing wave propagation in one dimension. The general solution of the equation may be written in the form $f(x+ct)+g(x-ct)$ where f and g are any suitably continuous functions: describe briefly what the two terms represent and identify the constant c. [3]

   Show, using complex exponentials or otherwise, that the disturbance $\cos(kx)\cos(\omega t)$ may be written in the form $f(x+ct)+g(x-ct)$ provided that there is an appropriate relationship between k and ω. [4]

2. Sketch the diffraction pattern formed at a large distance from an illuminated slit, and use it to explain (without detailed derivation) Rayleigh's criterion for the minimum angular separation between two images which can just be resolved. [3]

   For a circular aperture of diameter D the corresponding minimum angular separation using a wavelength λ is $1.22\lambda/D$. Deduce the maximum distance at which the human eye, with a pupil diameter of 4 mm, is capable of resolving the headlights of a car which are 1.22 m apart if the average wavelength of the light is 500 nm. [4]

3. State the change of phase which occurs when a light wave is reflected from (i) an optically more dense material, (ii) an optically less dense material. [2]

   Draw a diagram showing the formation of interference fringes between two glass slides which are in contact at one edge and separated by a human hair of diameter 10^{-1} mm 50 mm from that edge. Calculate the separation of successive bright fringes when they are produced by light of wavelength 500 nm. [5]

4. A sound wave travelling vertically downwards in air encounters the upper surface of water in a pool. State what boundary conditions must be applied at the interface in order to compute the reflection and transmission coefficients. [3]

   The coefficients of reflection and transmission for a wave incident normally on an interface between two materials with impedances Z_1 and Z_2 are

   $$r=\frac{Z_1-Z_2}{Z_1+Z_2} \quad \text{and} \quad t=\frac{2Z_1}{Z_1+Z_2},$$

   and the specific acoustic impedances of air and water are 400 kg m^{-2} s^{-1} and 1.45×10^6 kg m^{-2} s^{-1} respectively. If a sound wave of amplitude 10^{-6} m is incident from the air, calculate the reflected and transmitted amplitudes and hence verify that one of the boundary conditions is satisfied. [4]

5. An observer moves away, at a speed v_o, from a stationary source emitting a steady wave at frequency f. Show that the observed frequency is **[3]**

$$f' = f\left(1 - \frac{v_o}{c}\right),$$

where c is the wave speed in the medium.

If the source is also moving, away from the observer, with speed v_s, the observed frequency is

$$f' = f\frac{1 - v_o/c}{1 + v_s/c}.$$

If an ambulance drives up a beach towards a cliff at a speed of 33 m/s sounding its 1500 Hz siren, what will be the frequency of the reflected sound heard by the driver? The speed of sound in air is 330 m/s. **[4]**

6. Write down Snel's law of refraction, explaining all the symbols you use. **[2]**

A narrow beam of light is incident from air at an angle of 45 degrees on a parallel-sided slab of glass, 1 cm thick, with refractive index 1.5. Calculate the angle of the beam inside the slab. Draw a diagram showing the passage of the light through the slab, showing clearly the angle you have just calculated, and hence find the lateral (perpendicular to the beam direction) shift of the emerging beam. **[5]**

7. Draw a diagram showing an experimental arrangement for the observation of Young's fringes. **[3]**

Two loudspeakers placed 10 m apart are fed with identical 1320 Hz sinusoidal signals. An observer is 50 m away, equidistant from the two loudspeakers. Explain why the intensity heard by the observer decreases and then increases again as the observer moves sideways. Calculate distance the observer has moved when the maximum intensity is recovered. The speed of sound in air is 330 m/s. **[4]**

8. Draw ray diagrams to illustrate chromatic aberration and spherical aberration for a converging lens. **[4]**

Which of these aberrations would you *not* expect to occur with a similarly sized spherical mirror? Suggest how the lens system might be modified to reduce the effects of this aberration. **[3]**

SECTION B

9. Show that the speed of longitudinal waves on a rod of uniform cross-section with density ρ and Young's modulus Y is given by $v = \sqrt{Y/\rho}$. **[5]**

   Calculate the two lowest frequencies of standing longitudinal waves on a freely suspended steel rod of length 1 m. The density of steel is 7900 kg m^{-3} and its Young's modulus is 200 GN m^{-2}. **[5]**

   Derive an expression for the average kinetic energy density in a longitudinal wave of angular frequency ω and amplitude a in a rod of density ρ. Given that the average potential energy density density in the wave is equal to the average kinetic energy density, calculate the amplitude of the lowest longitudinal wave mode in the rod described above if the total energy in this mode is 0.03 J and the radius of the rod is 10 mm. **[10]**

10. Explain what is meant by the *principle of superposition* and by *dispersion*. By considering two waves, $y_1(x,t) = A\sin(k_1x - \omega_1 t)$ and $y_2(x,t) = A\sin(k_2x - \omega_2 t)$, explain the terms *carrier wave* and *envelope*, and derive expressions for the phase and group velocities of a dispersive wave. **[12]**

    The angular frequency ω of longitudinal waves with wavevector k on a chain of beads, each of mass m, joined together by springs with unstretched length a and spring constant κ is $\omega = \sqrt{\kappa/m}\sin(ka/2)$. Show that the phase and group velocities are equal in the limit of long waves, and calculate that limiting velocity if $m = 10$ gm, $a = 1$ cm, and $\kappa = 1$ N/m. **[6]**

    What is the group velocity of a wave with wavelength $2a$? **[2]**

11. Draw a diagram of a Michelson interferometer: include and explain the function of a compensating plate. **[5]**

    Show that if the interferometer is set up with equal optical path lengths along the two arms the variation of intensity as a function of angle away from the normal is

    $$I(\theta) = I(0)\sin^2\left(\frac{2\pi}{\lambda}d\cos(\theta)\right),$$

    explaining the significance of each of the terms. Hence describe the appearance of the fringe pattern when the mirrors are set up in a) a parallel and b) an oblique configuration. **[10]**

    A container 10 cm long with flat parallel windows in the ends is placed in one arm of the interferometer, and a fringe pattern is set up with light of wavelength 600 nm. All the air is pumped out of the cell and the fringe pattern shifts by 100 bright fringes. What was the refractive index of the air? **[5]**

12. Explain what is meant by the *coherence* of a wave pattern, distinguishing between longitudinal and transverse coherence. **[3]**

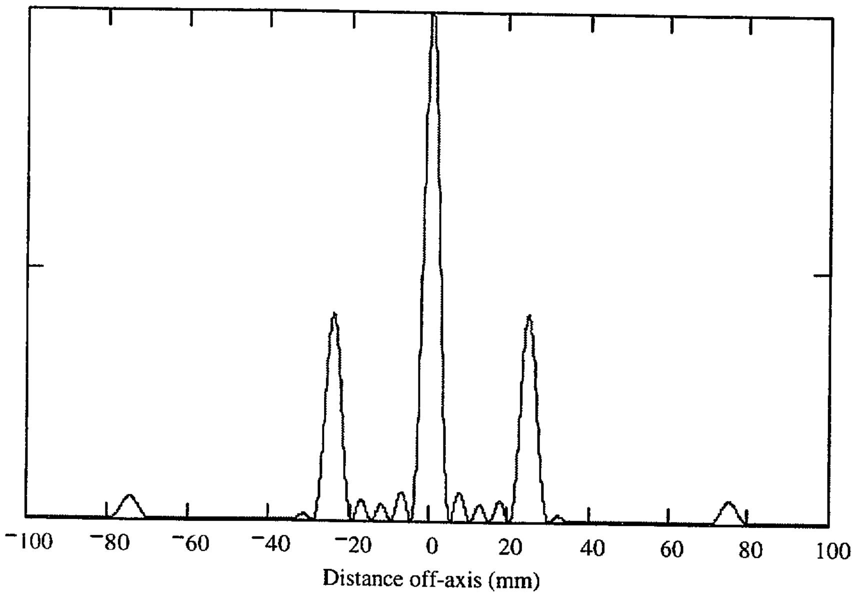
Show that the total electric field in the Fraunhofer diffraction pattern produced from monochromatic light with wavelength λ by a grating with N narrow slits at uniform spacing d is given by

$$E = E_1 \frac{1 - e^{2iN\gamma}}{1 - e^{2i\gamma}},$$

where E_1 is the field due to one slit and $\gamma = \pi d \sin(\theta)/\lambda$. Hence show that the intensity pattern may be written as **[5]**

$$I(\theta) = I(0) \left[\frac{\sin(N\gamma)}{N \sin(\gamma)} \right]^2 .$$

[3]



When the finite width w of the slits is taken into account the factor $I(0)$ may be written

$$I(0) = I_0 \left[\frac{\sin(\beta)}{\beta} \right]^2 ,$$

where $\beta = \pi w \sin(\theta)/\lambda$.

The figure shows the intensity pattern observed at a distance of 1 m from a grating illuminated by light of wavelength 500 nm. Deduce as much as you can about the structure of the grating. **[9]**

13. Draw a ray diagram to show the formation of a real image from a real object by a converging lens. If the focal length of the lens is 20 cm, how far from the lens must the object be placed if the image is to be magnified 10 times? **[8]**

The same lens is used as the objective in an compound lens, with a diverging lens of focal length 10 cm as the eyepiece 15 cm away from it. Calculate the positions of the focal points of this compound lens. Illustrate your calculation with appropriate ray diagrams. **[12]**

END OF PAPER