Mathematical Methods **Spring Term 2019**

Problem Sheet 9

1. Using the trapezium rule, evaluate

$$i)\ \int_0^1 e^{-x^2} dx \qquad ii)\ \int_0^1 \frac{dx}{1+x^4}$$

correct to 2 decimal places. (The error may be estimated by halving the mesh size and observing the resultant change in value given by the trapezium rule).

2. Using Simpson's rule, evaluate

$$i)\ \int_0^\pi \frac{\sin x}{x} dx \qquad ii)\ \int_0^1 \frac{x^2}{1+x^2} dx$$

correct to 3 decimal places. For ii) compare your result with the exact answer.

3. Show graphically that the equation $e^{-x} = \ln x$ has a solution with $1 < x < 2$. Hence solve the equation to 4 decimal places by the Newton-Raphson method.

4. Show graphically that the equation $e^{-x} = 2x^2 + \frac{1}{2}$ has three solutions which lie between -3 and 1. Solve the equation to 2 decimal places by the Newton-Raphson method.

5. Use the simple Runge-Kutta process

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2), \qquad k_1 = f(x_n, y_n), \qquad k_2 = f(x_{n+1}, y_n + hk_1)$$

to find $y(1)$ for the equation

$$\frac{dy}{dx} = f(x, y) = y$$

subject to $y(0) = 1$, choosing $h = 0.5$. Compare your answer with the exact solution. Repeat the calculation with $h = 0.25$. Work to three decimal places.

6. Use Runga-Kutta to find $y(0.2)$ for the equation

$$\frac{dy}{dx} = f(x, y) = 10x^2 + y$$

subject to $y(0) = 2$, choosing the interval $h = 0.1$. Work to 4 decimal places. Compare your answer with the exact solution.

7. i) Solve the following set of equations by Gaussian elimination

$$\begin{aligned} x + 2y - z &= 4 \\ 2x + y + z &= 5 \\ 2x - y + 2z &= 2. \end{aligned}$$

ii) Use the Gauss-Jordan technique to find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 2 & -1 & 2 \end{pmatrix}.$$