1B45 Mathematical Methods for Physics 1

Introduction

The 1B45 Mathematics course attempts to accommodate the changes in mathematical preparation afforded by A levels. These changes have become evident in the difficulties some of our students encounter in our physics and astronomy courses. Many of these difficulties stem from a loss of fluency in mathematics and associated manipulative skills. In physics and astronomy, laws are expressed mathematically, and mathematics is also needed to derive the consequences and interrelationships of these laws.

I urge you to accept the challenge of mastering this course because effort now will make it much easier to cope with the physics and astronomy courses. The challenge is as much to me as it is to you!!!

How are we to meet this challenge? Come to all the lectures and take a good set of notes. You must participate in the lectures and taking good notes is the start of this. I intend to develop the course in a reasonably logical and rigorous manner and to do many examples. The emphasis will be on the mathematics you will need for your physics and astronomy courses, and on acquiring fluency in mathematical manipulation. You really should go through your notes after every lecture, rewriting them as necessary, and going through everything I wrote down on the board.

Do the weekly problem sheets, normally given out in the Thursday lectures and collected in the Tuesday of the following week. **Three of these problem sheets will be devoted to simple manipulative skills in differentiation, integration and vectors. You will be required to achieve more than 80% in each of the three, repeating them as necessary before you can complete the course.**

Starting in a few weeks time there will be four bi-weekly two-hourly problem classes where groups of 15 - 20 students attempt a problem sheet. Demonstrators will circulate around to give help as needed. Again, don't be afraid to ask for help.

Half way through this term and in the last week of it there will be a class exam to tell me and you how well the course has gone. There is, of course, the final 1B45 exmination at the end of the academic year.

The book *Mathematical Methods for Physics and Engineering* , by K. F. Riley, M. P. Hobson and S. J. Bence, (3rd Edition - Cambridge), available from Dr Furniss, is a good text book for PHAS1245 and the follow on Maths courses PHAS1246 and PHAS2246 (2nd year).

Finally, do not underestimate this course. If you think you are familiar with most of the material, particularly early on in the course, persevere, there will be new material, and I will have some quite challenging problems to keep the most able of you happy! This course has been extremely successful with a good response from the students and a most satisfactory examination performance. Remember that a good effort now will make your physics and astronomy courses so much easier and much more fun!

Aims

- provide the mathematical foundations required for all the first semester and some of the second semester courses in the first year of the physics and astronomy programmes;
- prepare students for the second semester follow-on mathematics course PHYS1B46;
- give students practice in mathematical manipulation and problem solving.

Objectives

After completing this half-unit course, the student should be able to:

- solve simultaneous and quadratic equations with examples taken from physical situations;
- sum arithmetic, geometric and other simple series;
- appreciate the relation between powers, exponentials and logarithms and the more general concept of the inverse function in terms of a graphical approach;
- derive the values of the trigonometric functions for special angles;
- understand the relation between the hyperbolic and exponential functions;
- differentiate simple functions and apply the product and chain rules to evaluate the differentials of more complicated functions;
- find the positions of the stationary points of a function of a single variable and determine their nature;
- understand integration as the reverse of differentiation and to use this to evaluate integrals almost 'by inspection';
- evaluate integrals by using substitutions and integration by parts;
- understand a definite integral as an area under a curve;
- evaluate the Gaussian, Feynman, Gamma and Breit Wigner (Lorentzian) integrals and generate further definite integrals by differentiation w. r. t. a parameter;
- differentiate up to second order a function of 2 or 3 variables and be able to test when an expression is a perfect differential;
- change the independent variables by using the chain rule and, in particular, work with polar coordinates;
- find the stationary points of a function of two independent variables and to determine their nature;
- find the stationary points of function of two or more variables subject to constraints (Lagrange multipliers);
- manipulate real three-dimensional vectors, evaluate scalar and vector products, find the angle between two vectors in terms of components;
- construct vector equations for lines and planes and find the angles between them;
- express vectors, including velocity and acceleration, in terms of basis vectors in polar coordinate systems;
- understand the concept of convergence for an infinite series, be able to apply simple tests to investigate it;
- expand an arbitrary function of a single variable as a power series (Maclaurin and Taylor), make numerical estimates, and be able to apply L'Hopitals rule to evaluate the ratio of two singular expressions;
- represent complex numbers in Cartesian and polar form on an Argand diagram.

- perform algebraic manipulations with complex numbers, including finding powers and roots;
- apply de Moivres theorem to derive trigonometric identities and understand the relation between trigonometric and hyperbolic functions through the use of complex arguments. components;

Course Outline / Syllabus

40 lectures plus 6 problems classes.

- **Elementary Functions (mainly revision)**
  Manipulation of algebraic equations, powers, exponentials and logarithms, inverse functions, trigonometric functions, sine, cosine and tangent for special angles, hyperbolic functions.
- **Vectors**
  Definition, addition, subtraction, scalar and vector multiplication. Vector and scalar triple products, vector equations (Third order determinants only very briefly).
- **Differentiation (mainly revision)** Definition, product rule, function of a function rule, implicit functions, logarithmic derivative, parametric differentiation, maxima and minima.
- **Integration (mainly revision)**
  Integration as converse of differentiation, changing variables, integration by parts, partial fractions, trigonometric and other substitutions, definite integral, integral as the area under a curve, trapezium rule, integral of odd and even functions. The Gaussian, Feynman, Breit Wigner (Lorentzian) integrals. Further definite integrals obtained by differentiation w. r. t. a parameter.
- **Partial Differentiation**
  definition, surface representation of functions of two variables, total differentials, chain rule, change of variables, second order derivatives. Maxima, minima and saddle points for functions of two variables. Stationary values of functions subject to constraints.
- **More Vectors**
  Vector geometry - straight lines and planes. Vector differentiation, vectors in plane polar, cylindrical, and spherical polar coordinates.
- **Series**
  Summation of arithmetic, geometric and other simple series. Sequences and series, convergence of infinite series. Power series, radius of convergence, simple examples including the binomial series. Taylor and Maclaurin series, L'Hopital's rule.
- **Complex Numbers**
  Representation, addition, subtraction, multiplication, division, Cartesian, polar exponential forms, De Moivre's theorem, powers and roots, complex equations.