PHAS1245: Mathematical Methods I - Problem Sheet 3

(Solutions to be handed in at the lecture on Tuesday 23th October 2007)

Staple your answer sheets together and put **your name** and your **tutor's name** on your script (or Dr. Konstantinidis, if you have no tutor in the P&A department).

1. Evaluate [3]
$$\int \cos^2 x \, dx \,,$$
using two different methods.

2. Evaluate [3]
$$\int \cos^4 x \, dx \,.$$

3. Evaluate [3]
$$\int \frac{x^3}{(x+1)(x-3)} dx \,,$$
using the trick with partial fractions (hint: look at your notes from week 1).

4. Evaluate [4]
$$\int \ln x \, dx \qquad \text{and} \qquad \int (\ln x)^2 \, dx \,.$$

5. Evaluate [3]
$$\int x \, e^{-ax^2} \, dx \,.$$

6. Show that [4]
$$\int_0^\infty x^3 \, e^{-ax^2} \, dx = \frac{1}{2a^2} \,.$$
(Hint: start from the previous integral and integrate by parts). This form of integral comes up in the evaluation of the average speed of molecules of a gas, based on the Maxwell-Boltzmann distribution.