PHAS1245: Mathematical Methods I - Problem Class 2

Week starting Monday 15th October

1. Differentiation using the product and/or the chain rule.

   (a)

$$\text{If } \; y = \left(x + \frac{1}{x}\right)^2 \; \text{ show that } \; \frac{dy}{dx} = 2x - \frac{2}{x^3} \; .$$

   (b) Obtain

$$\frac{dy}{dx} \; \text{ if } \; y = \sqrt{x} + \frac{1}{\sqrt{x}} \; .$$

   (c)

$$\text{If } \; y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \; \text{ show that } \; \frac{dy}{dx} = \frac{1}{\sqrt{x}} \frac{-1}{(\sqrt{x}-1)^2} \; .$$

   (d) Obtain

$$\frac{dy}{dx} \; \text{ if } \; y = sin^{-1} x^2 \; .$$

   (e)

$$\text{If } \; y = ln \, \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}-\sqrt{x}} \; \text{ show that } \; \frac{dy}{dx} = \frac{\sqrt{a}}{\sqrt{x}(a-x)} \; .$$

2. Given the curves $f(x) = x^2$ and $g(x) = 1 - x^2$, find the angle between them at their points for interception.

3. The parametric equations for the motion of a charged particle released from rest in electric and magnetic fields at right angles to eachother take the forms

$$z = a(\theta - \sin\theta) \qquad\qquad y = a(1 - \cos\theta) \, .$$

   Show that the tangent to the curve has slope $\cot(\theta/2)$. Use this result at calculated values of x and y to sketch the form of the particles trajectory.

4. Show how the number of stationary points of the function

$$y(x) = x a^{2x} \mathrm{e}^{x^2} \qquad\qquad (a > 0)$$

   depends on the value of a and in the case where only one stationary point exists, determine that point.