PHYS1B28: Thermal Physics
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Problem Sheet 1 (2005) – Model answers

1. An ideal gas exerts a pressure of 1.52 MPa when its temperature is 25 ^{o}C and its volume is 10^{-2} m^3.
   (a) How many moles of gas are there?
   (b) What is the mass density if the gas is molecular oxygen, H_2?
   (c) What is the mass density if the gas is oxygen, O_2?

(a) To find the number of moles use the ideal gas law $PV = nRT$. Thus

$$n = \frac{PV}{RT} = \frac{(1.52\times10^{6})(10^{-2})}{(8.31)(298.15)} = 6.135 \text{ mol} ✓✓$$

(b) The atomic mass of hydrogen is 1.008. Hence 1 mol of hydrogen gas, H_2, contains M = 2.016 g = 2.016×10^{-3} kg. The mass density of hydrogen is then

$$\rho = \frac{nM}{V} = \frac{(6.13)(2.016\times10^{-3})}{10^{-2}} = 1.24 \text{ kg/m}^3 ✓✓$$

(c) The atomic mass of oxygen is 16, so that 1 mol of oxygen contains M = 32 g = 32×10^{-3} kg. The mass density of oxygen is:

$$\rho = \frac{nM}{V} = \frac{(6.13)(32\times10^{-3})}{10^{-2}} = 19.6 \text{ kg/m}^3 ✓$$

[5]

2. (a) The average kinetic energy of the molecules of the ideal gas at 15 ^{o}C has the value of KE. At what temperature will the average kinetic energy of the same gas double this value, i.e. become 2 KE?

Average kinetic energy of molecules in a gas is $\overline{ke} = \frac{3}{2}k_B T$, i.e. is proportional to temperature T. ✓

T = 15 + 273.15 = 288.15 K✓

2 KE should correspond to 2T = 576.3 K ✓ = 303.15 ^{o}C.

(b) Find a root-mean-square (RMS) speed of a mist particle of mass 1×10-15 kg would have at room temperature according to kinetic theory.

$$\sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} ✓ = \sqrt{\frac{3\times1.38\times10^{-23}\times298}{1\times10^{-15}}} = 3.5\times10^{-3} \text{ m/s.} ✓$$

[5]

3. A beam of particles, each of mass m and speed υ, is directed along the *x* axis. The beam strikes the area of 1 mm^2 with 10×10^{15} particles striking per second. Find the pressure on the area due to the beam if the particles stick to the area when they hit. Evaluate for an electron beam in a television tube where $m = 9.1\times10^{-31}$ kg and $\upsilon = 8\times10^{7}$ m/s.

The momentum change due to collision of each particle with the wall is $\Delta p = m \times \upsilon$ because particles stick to the wall. ✓

$$P = \frac{F}{A} ✓ = \frac{\Delta p}{\Delta t \times A} N$$, where N is the number of particles colliding with the wall per second ✓

The area $A = 1\ mm^2 = 10^{-6}\ m^2$. Therefore $P = (10\times10^{15}/10^{-6}) = 10^{22}\ m\times\upsilon$. ✓

Calculating the electron momentum $(9.1\times10^{-31})(8\times10^{7})$, we find $P = 0.73$ Pa ✓

[5]

4. The pressure of a gas in a 100 mL container is 200 kPa and the average translational kinetic energy of gas particles is 6.0×10^{-23} J. Find the number of particles in the container. How many moles of gas are in the container?

From kinetic theory

$PV = \frac{2}{3}\overline{ke} \times N$,✓ where $\overline{ke}$ is the average kinetic energy of gas molecules, and N is the total number of molecules. ✓

Therefore $N = \frac{3}{2}\frac{PV}{\overline{ke}} = \frac{3}{2}\frac{(2.0\times10^{5})(100\times10^{-6})}{(6.0\times10^{-23})} = 5.0\times10^{23}$ molecules. ✓

Number of moles $n = N/N_A$, ✓where N_A is the Avogadro number. $n = 0.83$ mol. ✓

[5]

5. A piston pump having a barrel of effective volume 100 cm^3 is used to exhaust a vessel of volume 2.5×10^{3} cm^3. If the initial pressure of the air in the vessel is 1 atm, estimate the reduction in pressure produced by two strokes of the pump, assuming that all processes are isothermal and that air behaves as an ideal gas. Further, estimate the number of strokes needed to reduce the pressure the pressure in the vessel from 1 atm to 0.02 atm.

Since the process is isothermal, one can apply the Boyle's law $P_0V_0 = P_1V_1$. ✓

If P_0 is the initial pressure in the vessel of volume V, and v is the volume of pump barrel, after the first stroke the pressure P_1 is given by equation:

$P_0V = P_1(V + v)$ ✓ and $P_1 = \frac{P_0V}{V+v}$ ✓

Then after the second stroke:

$P_1V = P_2(V + v)$ ✓ or $\frac{P_0V}{V+v}V = P_2(V+v)$. ✓ Thus $P_2 = \frac{P_0V^2}{(V+v)^2}$. ✓

After n strokes the pressure P_n is therefore given by:

$$P_n = \frac{P_0V^n}{(V+v)^n}\,. \checkmark$$

Therefore after the two strokes the pressure is 0.925 atm. ✓

To find the number of strokes one needs to write:

$$\frac{P_n}{P_0} = \left[\frac{V}{V+v}\right]^n \text{, thus } \ln\frac{P_n}{P_0} = n \times \ln\left[\frac{V}{V+v}\right] \checkmark \text{ and } n = \frac{\ln\frac{P_n}{P_0}}{\ln\left[\frac{V}{V+v}\right]} = \frac{\ln(0.02)}{\ln(0.96)} = 98. \checkmark$$

So around 100 strokes are needed to reduce the pressure in the vessel from 1 atm to 0.02 atm.

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