RELATIVITY (MTH6132)

SOLUTIONS TO THE PROBLEM SET 2

1. Only the second and the third expressions make sense. The first one has free indices not matching. In the third expression the index k appears as a dummy on one side and free on the other. In the fifth expression k appears repeated three times.

2. As all inertial systems of reference are equivalent, the inverse expressions have to have the same functional form. The only thing to be remembered is that if F' moves away from F with velocity v, then F' sees F moving away with velocity $-v$. Accordingly, th required expressions read:

$$V = \frac{V' + v}{1 + vV'/c^2}, \qquad a = \frac{a'}{\gamma^3(1 + vV'/c^2)^3},$$

3. One has that

$$\begin{aligned}
G_{ij}: &\quad G_{11}, G_{12}, G_{21}, G_{22},\\
A^i B_i: &\quad A^1 B_1 + A^2 B_2,\\
\Gamma^i{}_{jk}: &\quad \Gamma^1{}_{11}, \Gamma^1{}_{12}, \Gamma^1{}_{21}, \Gamma^1{}_{22}, \Gamma^2{}_{11}, \Gamma^2{}_{12}, \Gamma^2{}_{21}, \Gamma^2{}_{22},\\
\Gamma^i{}_{ij}: &\quad \Gamma^1{}_{11} + \Gamma^2{}_{21}, \Gamma^1{}_{12} + \Gamma^2{}_{22},\\
R^i{}_{jkl}: &\quad R^1{}_{111}, R^1{}_{112}, R^1{}_{121}, R^1{}_{122},\\
&\quad R^2{}_{111}, R^2{}_{112}, R^2{}_{121}, R^2{}_{122},\\
&\quad R^1{}_{211}, R^1{}_{212}, R^1{}_{221}, R^1{}_{222},\\
&\quad R^2{}_{211}, R^2{}_{212}, R^2{}_{221}, R^2{}_{222}.
\end{aligned}$$

4. The train has lenght $\Delta x' = L$ in its own frame (F'). From the lectures one has that $\Delta x' = \gamma \Delta x$. For an observer in F, $\Delta x = L/3$. Hence

$$L/3 = L\left(1 - \frac{v^2}{c^2}\right)^{1/2}.$$

Solving for v one finds

$$v/c = 2\sqrt{2}/3.$$