Course Work 6

Q1.

a) A binary system consists of two neutron stars of the same mass M. The orbital period of the system is P. Using Newtonian mechanics, estimate to an order of magnitude the separation between the neutron stars, r, and the fractional relativistic corrections to the orbital motion.

b) Evaluate the relativistic corrections if $P = 8$ min and $M = 1.5M_{\odot}$. Compare your estimate with relativistic effects in the solar system. It is known that the perihelion shift of Mercury is 43" per century. What analogous shift can you expect in the case of the binary system of neutron stars? (Hint: The relativistic shift per one orbital period is of order r_g/r, where r_g is gravitational radius of the neutron star.)

Q2.

The quadrupole formula for the metric perturbation associated with gravitational waves is given by

$$h_{\alpha\beta} = -\frac{2G}{3c^4R}\frac{d^2D_{\alpha\beta}}{dt^2}(t - R/c),$$

where R is the distance to the source of the gravitational waves and

$$D_{\alpha\beta} = \int (3x_\alpha x_\beta - r^2\delta_{\alpha\beta})dM$$

is the quadrupole tensor of the source. Consider a mass m moving along circular orbit around the black hole of mass M, assuming that $m \ll M$.

a) Show that all the amplitudes $h_{\alpha\beta}$ of gravitational wave, emitted by such system, are periodic functions of time with $\omega = 2\omega_0$, where $\omega_0 = 2\pi/T$, and T is the orbital period.

b) Show that, to an order of magnitude (omitting the indices α and β)

$$h \approx \frac{r_g}{R}\left(\frac{R_g\omega}{c}\right)^{2/3},$$

where r_g is the gravitational radius of the mass m and R_g is the gravitational radius of the black hole.

Q3.

The future LISA mission will be able to detect gravitational waves with $h > 10^{-23}$, if $10^{-4}Hz < \omega < 3 \cdot 10^{-3}Hz$. From what distance will it be possible to detect gravitational radiation from the binary system, containing the black hole of mass $m = 3M_{\odot}$, moving along a circular orbit with radius $r = 10^4R_g$ around the massive black hole of mass $M = 10^3M_{\odot}$?