Relativistic Astrophysics. 2009. Course Work 4

Q1.

Write a brief essay (1-2 Pages) "Why invisible black holes can be seen in astronomical observations".

Q2.

Explain briefly what is the main difference between the limit of stationarity and the event horizon of a black hole?

Q3.

Given that n_i is a covariant vector and

$$n^i = g^{ik} n_k.$$

Taking into account that g_{ik} and g^{ik} are reciprocal to each other, show that

$$g^{ik} n_i n_k = g_{ik} n^i n^k.$$

Q4.

Let the interval ds is given as

$$ds^2 = \left(1 - \frac{A}{x^1}\right)(dx^0)^2 - \left(1 - \frac{4A^2}{(x^1)^2}\right)^{-1}(dx^1)^2 - (dx^2)^2 - (dx^3)^2,$$

where A is a constant.

a) Find g^{11}.

b) Show that this interval corresponds to a space-time geometry with the limit of stationarity and the event horizon. Determine the position of both these surfaces.

Q5.

a) Discuss briefly what is the significant deference between the "Laplacian black hole" and the black hole in General Relativity.

b) Explain why the surface $r = r_g$ in the Schwarzschild metric is the event horizon. Where the limit of stationarity is located?. Show that the surface $r = r_g$ is a null surface.

Q6.

Consider a rotating black hole described by the Kerr metric.

a) What is meant by the event horizon, the "limit of stationarity" and the "ergosphere"? (Compare with the case of the Schwarzschild black hole).

b) Describe briefly the Penrose process of extraction of energy from a rotating black hole and explain why this mechanism does not contradict to the statement, that nothing can escape from within black hole.

Q7.

a) Give the definition of the Ricci tensor R_{ik} and prove that

$$R_{ik} = \frac{\partial \Gamma^l_{ik}}{\partial x^l} - \frac{\partial \Gamma^l_{il}}{\partial x^k} + \Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{km}$$

b) Starting from the Einstein equations in the form

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik},$$

where G is the gravitational constant, prove that

$$T^i_k = \frac{c^4}{8\pi G} \left(R^i_k - \frac{1}{2} \delta^i_k R \right).$$

c) What can you say about the nature of gravitational field, for which $R_{ik} = 0$, while R_{ikln} is not equal to zero?