

## Relativistic Astrophysics. 2009. Course Work 3

### Q1.

- a) Explain what is the reciprocal tensor.
- b) Demonstrate how using the reciprocal contravariant metric tensor  $g^{ik}$  and the covariant metric tensor  $g_{ik}$  you can form contravariant tensor from covariant tensors and vice versa.
- c) Show that in an arbitrary non-inertial frame

$$g^{ik} = S_{(0)0}^i S_{(0)0}^k - S_{(0)1}^i S_{(0)1}^k - S_{(0)2}^i S_{(0)2}^k - S_{(0)3}^i S_{(0)3}^k,$$

where  $S_{(0)k}^i$  is the transformation matrix from locally inertial frame of reference (galilean frame) to this non-inertial frame.

### Q2.

- a) Give a rigorous proof that the interval is a scalar.
- b) Prove that the metric tensor is symmetric.

**Q3.** Using lecture notes 3, write a short essay (1-2 pages) "Proper time and physical distances".

### Q4.

- a) Show that all covariant derivatives of metric tensor are equal to zero.
- b) Find the the relationship between the Cristoffel symbols and first partial derivative of the metric tensor.