Relativistic Astrophysics. 2009. Course Work 2. Solutions

Q1.

Formulate the equivalence principle and explain what is the difference in interpretation of this principle in Newtonian theory and in General relativity.

This principle states that an uniform gravitational field is equivalent to a uniform acceleration of reference frame.

In Newton theory the motion of a test particle is determined by the following equation of motion

$$m_{in}\vec{a} = -m_{gr}\nabla\phi,$$

where $\vec{a}$ is the acceleration of the test particle, ϕ is newtonian potential of gravitational field, m_{in} is the inertial mass of the test particle and m_{gr} is its gravitational mass. The fact that all test particles move with the same acceleration for given ϕ is explained within frame of newtonian theory just by the following "coincidence":

$$\frac{m_{in}}{m_g} = 1,$$

i.e. inertial mass m_{in} is equal to gravitational mass m_{gr}.

The General Relativity gives very simple and natural explanation of the Principle of Equivalence: In curved space-time all bodies move along geodesics, that is why their world lines are the same in given gravitational field. The situation is the same as in flat space-time when free particles move along straight lines which are geodesics in flat space-time. What is geodesic we will also discuss the next lecture.

Q2.

A rocket moves very far from any gravitating bodies with acceleration $5g$. Using the equivalence principle, show, that in first order with respect to h/R the redshift of a photon emitted at the bottom of the rocket and detected at its top is the same as if the rocket were at rest on the surface of a planet with mass M and radius R related by the following relationship: $MR_\oplus^2 = 5M_\oplus R^2$. Calculate the redshift if the height of the rocket is $169m$. (You can assume that the diameter of the Earth is 13 000 km and its gravitational radius is 1 cm).

The non-inertial frame of reference moving with acceleration $5g$ is equivalent to gravitational field of mass M and radius R if

$$\frac{GM}{R^2} = 5g = \frac{5GM_\oplus}{R_\oplus^2},$$

hence

$$MR_\oplus^2 = 5M_\oplus R^2.$$

From solution to question Q2 of CW1 we know that the redshift is

$$z = \frac{\delta U}{c^2},$$

where

$$U = -\frac{GM}{R}$$

and

$$\delta U = \delta R\frac{dU}{dR} = h\frac{GM}{R^2};$$

hence

$$z = \frac{GMh}{R^2c^2} = \frac{5GM_{\oplus}h}{R_{\oplus}^2} = \frac{5r_{\oplus g}h}{R_{\oplus}^2} \approx \frac{5 \cdot 10^{-2}m \cdot 169m}{(1310^6 m)^2} = 5 \cdot 10^{-14}.$$

Q3.

Formulate the covariance principle and explain the relationship between this principle and the principle of equivalence.

This principle says: The shape of all physical equations should be the same in an arbitrary frame of reference, including the most general case of non-inertial frames. If in contrast to the covariance principle the shape of physical equations were different in local inertial frames in presence of gravitational field and in non-inertial frames in absence of gravitational field then these equations would give different solutions, i.e. different predictions for (a) standing on the Earth, feeling the effects of gravity as a downward pull and (b) standing in a very smooth elevator that is accelerating upwards with the acceleration g , hence these equations would contradict to the basic postulate of the General Relativity, the principle of equivalence, which states that a uniform gravitational field (like that near the Earth) is equivalent to a uniform acceleration. Hence, the covariance principle is the mathematical formulation of the principle of equivalence.