Relativistic Astrophysics. 2009. Course Work 1

Q1.

a) Following the original Laplace calculation within the framework of Newtonian gravity, show that the escape velocity from the surface of a gravitating body of mass M is equal to the speed of light, if the radius of the body is equal to $2GM/c^2$ (gravitational radius).

b) A star forms a black hole of mass M. Show that to an order of magnitude its density at the moment immediately before the formation of the black hole is

$$2 \times 10^{16} \; g \cdot cm^{-3} \left(\frac{M}{M_{\odot}} \right)^{-2} .$$

For what mass is this equal to the density of air ($\approx 10^{-5} g/cm^3$)?

Q2.

a) Consider a photon with energy $E = h\nu$ climbing out of a gravitational field and use energy conservation law to show that in traveling through a potential difference $\delta U \ll c^2$, the photon should experience a fractional frequency shift

$$z = -\frac{\delta U}{c^2} .$$

b) From observations of some unknown object it was found that a fractional frequency shift of spectral lines was equal to

$$z = \frac{|\delta \nu|}{\nu} = (3.2 \pm 1.2) \cdot 10^{-4} .$$

Assuming that this redshift is explained by the redshift in the gravitational field of a solar mass object, calculate the predictable redshifts caused by a star of solar type, a white dwarf and a neutron star, whose typical radii may be taken to be 700000 km, 6000 km and 10 km, respectively. Hence determine which type of object was observed.

Q3.

a) Using simple Newtonian estimates, show that the radius of tidal disruption, R_{TD}, for a star of mass m and radius r in the gravitational field of the black hole of mass M is

$$R_{TD} \approx r \left(\frac{M}{m} \right)^{1/3} .$$

b) Find the critical value of the black hole mass, M_{crit}, for which R_{TD} equals the gravitational radius of the black hole.