2B27 Problem Sheet 3

1. Show that the maximum power of the wind blowing with velocity v at an angle θ with respect to the normal of a wind turbine, rotor diameter d can expressed as

$$P = \frac{1}{8}\rho\pi d^2 v^3 \cos^3\theta.$$

[5]

Briefly explain why the maximum obtainable power from the wind on a turbine which is 100% efficient must be less than this. **[3]**

Write down an expression for the coefficient of performance and show that it has a maximum of 16/27. **[2]**

2. An estuary has area 100 km^2, width 0.5 km and depth at low tide 25 m. If the tidal range is 6 m, calculate the average power that could be obtained from the estuary using a barrage scheme. **[5]**

Calculate the average flow rate during the rising tide. **[3]**

Calculate the power that could be obtained from the average rising tidal flow using a 100% efficient underwater turbine with blade radius 8 m. Why would the average power during the rising tide be higher than this? **[6]**

How many turbines would be required to power a small town requiring 20 MW. **[1]**

3. A small block of flats has side lengths 10 m and 5 m and the rooms on each floor are 2 m high. All the flats are kept at the same temperature so heat loss through the ceiling and floor of the middle flat can be neglected. If the overall U value for the walls of the flats is 2 W m^{-2}K^{-1} and the input and output boundary layer resistances h are 0.1 K m^{2}W^{-1} calculate the steady state temperature of the middle flat if the flat is heated by a 1 kW heater and the outside temperature is 0° C. **[5]**

A building with surface area A and volume V has an overall thermal transmittance (including boundary resistance) of U. The outside temperature varies with time t as

$$T_{\text{out}} = T_0 \exp(-kt)$$

where T_0 is a constant. Derive the following equation determining the evolution of the temperature, T, inside the building.

$$T + \frac{\rho V c_v}{AU}\frac{dT}{dt} = T_0 \exp(-kt).$$

[4]

Solve this differential equation to show that the temperature can be expressed as

$$T = \frac{T_0}{AU - k\rho V c_v}\left[k\rho V c_v \exp\left(\frac{AU}{\rho V c_v}\right) + AU\exp(-kt)\right].$$

[11]