Quantum Physics – Homework 8

Due Thursday 15th March at 4. Attempt answers to all questions.

Hand in your script by the deadline into the post box near the secretaries' offices on level 1. **Assignments handed in past the deadline will not be marked.** Course title, week number and student name should appear on every sheet of the worked exercises, which should be securely bound together. Please *also* write your tutor's initials and time of tutorial on the cover sheet.

Problem 1 `[20 marks]`

Consider a particle which can move on a line segment parameterised by x for $0 \leq x \leq 1$, and let $\psi(x) = a\,x\,(x-b)$ be its wavefunction. Here a and b are constants.

(i) Determine b such that $\psi(x)$ obeys the correct boundary conditions for a particle constrained to move for $0 \leq x \leq 1$. **[3]**

(ii) Normalise the wavefunction. **[4]**

(iii) Calculate the expectation value $\langle x \rangle$ of x in the state described by the wavefunction $\psi(x)$ determined earlier. **[4]**

(iv) Calculate the uncertainty Δx of a measurement of x in the state ψ. This is defined as

$$\Delta x \;:=\; \Big(\langle x^2 \rangle \;-\; \langle x \rangle^2\Big)^{\frac{1}{2}} \;,$$

where the expectation values are evaluated in the state described by ψ. **[5]**

(v) Calculate the expectation value of the momentum of the particle. This is defined as

$$\langle p \rangle \;=\; \int_{\mathcal{D}} dx\, \psi^*(x) \Big(-i\hbar \frac{d}{dx} \Big) \psi(x) \;,$$

where the integral is extended to the domain $\mathcal{D}$ where the particle can move (in this case $\mathcal{D}$ is the line interval $0 \leq x \leq 1$). **[4]**

Problem 2 `[30 marks]`

A particle of mass m is free to move in the one-dimensional box defined by $0 \leq x \leq L$. At the time $t = 0$ its quantum state is described by the wavefunction

$$\psi(x) \;=\; N \sqrt{\frac{2}{L}} \left(\sin \frac{\pi x}{L} + \sin \frac{2\pi x}{L} \right) \;.$$

(i) Rewrite the wavefunction in terms of stationary states and determine the normalisation constant N such that the wavefunction is normalised. **[7]**

(ii) Calculate the expectation value of the position of the particle. You may find the following integral useful:

$$\int_0^{\pi} dz\, z \sin z \sin(2z) \;=\; -\frac{8}{9}\,.$$

[9]

(iii) What are the possible outcomes of a measurement of the energy of the particle, and what the corresponding probabilities? **[4]**

(iv) Write the wavefunction of the particle for $t > 0$. **[5]**

(v) At the time $t = t_0$ the energy is measured and found to be equal to $2\hbar^2\pi^2/(mL^2)$. Write down the wavefunction for $t > t_0$. **[5]**