Quantum Physics – Homework 7

Due Thursday 8^{th} March at 4. Attempt answers to all questions.

Hand in your script by the deadline into the post box near the secretaries' offices on level 1. **Assignments handed in past the deadline will not be marked.** Course title, week number and student name should appear on every sheet of the worked exercises, which should be securely bound together. Please *also* write your tutor's initials and time of tutorial on the cover sheet.

Problem 1 `[25 marks]`

A particle is constrained to move along the line segment parameterised by x for $0 \leq x \leq 2$. Let $\psi(x) = a\, x\, (x-2)$ be its wavefunction, where a is a constant.

(i) Determine a such that $\psi(x)$ is appropriately normalised. **[5]**

(ii) Determine the expectation value $\langle x \rangle$ of x in the quantum state given by the wavefunction ψ. This is defined as

$$\langle x \rangle_\psi \;:=\; \int_{\mathcal{D}} dx\, \psi^*(x)\, x\, \psi(x) \ ,$$

where $\mathcal{D}$ is the domain where the particle can move (here it is simply the line interval $0 \leq x \leq 2$), and $*$ denotes complex conjugation. **[6]**

(iii) Determine the most probable value of x.
Hint: Recall the statistical interpretation of the wavefunction, namely that the probability of finding the particle between x and $x + dx$ is $|\psi(x)|^2 dx$. **[6]**

(iv) What is the probability of finding the particle for $0 < x < 1/2$? and what is the probability of finding it for $1/2 < x < 2$?
Hint: Same hint as in part (iii). **[8]**

Problem 2 `[25 marks]`

1. State which of the functions below can be normalised in the interval specified in each case. **2.** For the case where the wavefunction can be normalised, find the explicit expression of the normalised wavefunction. **3.** For all normalised wavefunctions, find also the expectation values of the position, defined as usual as

$$\langle x \rangle \;:=\; \int_{\mathcal{D}} dx\, \psi^*(x)\, x\, \psi(x) \ ,$$

where $\psi(x)$ is the *normalised* wavefunction.

Here is the list of functions:

(i) $f_1(x) = x^2\sqrt{1-x}$ in $0 \leq x \leq 1$.

(ii) $f_2(x) = e^{3x^2}$ for $-\infty < x < \infty$.

(iii) $f_3(x) = e^{-(a/2)x^2}$ for $-\infty < x < \infty$. Here a is a positive number.

(iv) $f_4(x) = \sqrt{-x \log x}$, for $0 \leq x \leq 1$.

(v) $f_5(x) = (x^2+7)^{\frac{2}{5}}$, for $-\infty < x < \infty$.

Useful constants: Planck constant $h = 6.62 \times 10^{-34}$ J s. Boltzmann constant $k = 1.38 \times 10^{-23}$J/K. Speed of light $c = 3 \times 10^8$m/s. Electron mass: $m_e \simeq 0.5$ Mev/c^2.