Quantum Physics – Homework 6

Due Thursday 1st March 2012 at 4. Attempt answers to all questions.

Hand in your script by the deadline into the post box near the secretaries' offices on level 1. **Assignments handed in past the deadline will not be marked.** Course title, week number and student name should appear on every sheet of the worked exercises, which should be securely bound together. Please *also* write your tutor's initials and time of tutorial on the cover sheet.

Revision questions of the material studied so far `[32 marks]`

A1. Show that the non-relativistic approximation is not appropriate for a muon of kinetic energy 100 MeV (the mass of the muon is approximatively 200 times the mass of the electron). [4]

A2. Draw two graphs showing how the emissive power $R(\lambda, T)$ of a blackbody in thermal equilibrium depends on the wavelength: in the first graph draw the quantum mechanical result (Planck's law), in the second the classical (Rayleigh-Jeans) prediction. [4]

A3. Two narrow slits separated by $1.5\ 10^{-3}$m are irradiated with monochromatic coherent radiation. The spacing of the bright fringes observed on a screen parallel to the two slits and at a distance of 10 m from the slits is found to be $4\ 10^{-3}$ m. Find the wavelength of the light. [4]

A4. Consider the experimental apparatus of the photoelectric effect. Explain *briefly* why the stopping potential depends on the frequency but not on the intensity of the incident light. [4]

A5. A sample of cesium is illuminated with monochromatic light of wavelength equal to $2.5\ 10^{-7}$ m. Knowing that the work function for cesium is 1.9 eV, find the maximum kinetic energy of the emitted electrons. [4]

A6. In an electron microscope, electrons of de Broglie wavelength 0.1 nm are used. Find the kinetic energy of these electrons. [4]

A7. A plane wave wavefunction for an electron can be written in the form $\psi(x,t) = A \exp\left[i(kx - \omega t)\right]$. Write down the two de Broglie relations for k and ω, and use them to rewrite this expression in terms of the energy and momentum of the plane wave. [4]

A8. Explain what is meant by normalising the wavefunction. Explain why e^{ax} cannot be the wavefunction of a particle moving along the x-axis from $-\infty$ to $+\infty$. Here a is a real number. [4]

Problem 1 `[18 marks]`

Consider an experimental arrangement similar to that of the double-slit experiment, except that now we have N narrow slits instead of just two. It is illuminated by monochromatic, coherent light of wavelength λ. The separation between the slits is d, and you can neglect the diffraction effects due to the size of a single slit, which we take to be very narrow. An interference pattern is observed on a screen parallel to the slits at a very large distance D from the screen. The scope of this problem is to determine this diffraction pattern. More precisely:

(i) Firstly, write down the difference of phase of the (plane) waves produced by two adjacent slits (i.e. by two points at a separation equal to d), as done in class. In the experimental conditions $D \gg Nd$ we will take this difference to be the same between any pairs of adjacent slits. **[3]**

(ii) Next write down the combined electric field, i.e. the sum of all the contributions arising from the N slits. You may find the following formula useful:

$$\sum_{n=0}^{N-1} \cos(A + nB) = \cos\left[\frac{1}{2}(2A - B + BN)\right] \cdot \frac{\sin\frac{BN}{2}}{\sin\frac{B}{2}} .$$

In case you prefer to use a complex representation of the electric fields, the following result is useful:

$$\sum_{n=0}^{N-1} e^{inA} = \frac{e^{iNA} - 1}{e^{iA} - 1} .$$

[5]

(iii) Next, compute the time-averaged intensity (i.e. the time-average over one period) up to an overall normalisation, similarly to what we did in class for the double slit case. **[5]**

(iv) Finally, sketch the resulting diffraction pattern for the intensity as a function of $\sin\theta$ for the case N=7 (you may use any programme such as *Mathematica*). **[5]**

Useful constants: Planck constant $h = 6.62 \times 10^{-34}$ J s. Boltzmann constant $k = 1.38 \times 10^{-23}$J/K. Speed of light $c = 3 \times 10^{8}$m/s. Electron mass: $m_e \simeq 0.5$ Mev/c^2. $\hbar c \sim 197$ MeV $\times$ fermi.