Quantum Physics – Homework 5

Due Thursday 16$^{\text{th}}$ February 2012 at 4. Attempt answers to all questions.

Hand in your script by the deadline into the post box near the secretaries' offices on level 1. **Assignments handed in past the deadline will not be marked.** Course title, week number and student name should appear on every sheet of the worked exercises, which should be securely bound together. Please *also* write your tutor's initials and time of tutorial on the cover sheet.

Quick part A type questions `[20 marks]`

(i) Consider an electron of kinetic energy 1 eV. Calculate its de Broglie wavelength. Justify with an explicit calculation whether you can used a non-relativistic approximation or not. **[5]**

(ii) Consider an electron of kinetic energy 1 GeV. Calculate its de Broglie wavelength. Justify with an explicit calculation whether you can used a non-relativistic approximation or not. **[5]**

(iii) Consider a bullet of mass 150 g and speed 320 m/s. Calculate its de Broglie wavelength. **[5]**

(iv) Consider an idealised single-slit experiment performed with electrons of de Broglie wavelength λ. The length of the slit is a. What condition should λ obey in order to have diffraction? **[5]**

Problem 1 `[20 marks]`

Consider the Compton scattering of a beam of X-ray photons of wavelength $\lambda = 0.1$ nm by a free electron which is initially at rest.

(i) Write down the momentum and energy conservation equations. **[5]**

(ii) Prove that the angle ϕ between the direction of the scattered electron and the incident beam is related to the angle θ formed by the direction of the outgoing photons with the direction of the incident beam by the following equation

$$\tan\phi = \left[\tan\frac{\theta}{2}\left(1 + \frac{E}{mc^2}\right)\right]^{-1},$$

where E is the energy of the incident photon. **[15]**

Problem 2 `[10 marks]`

Consider a single slit of length $a = 0.01$ mm. Light of wavelength $\lambda = 500$ nm is diffracted through the slit. Find the separation of the first two minima on a screen parallel to the slits, at a distance $D = 1$ m from the slits. **[10]**

Optional

We saw in class that the diffraction + interference figure from a double-slit experiment (with a = size on one slit, d = separation between the slits, $d \gg a$) is expressed by the following curve:

$$I(\theta) \;\sim\; \frac{\sin^2\left(\pi\frac{a}{\lambda}\sin\theta\right)}{\left(\pi\frac{a}{\lambda}\sin\theta\right)^2}\left[1+\cos\left(2\pi\frac{d}{\lambda}\sin\theta\right)\right] .$$

Let me set $f(x) := (\sin^2 x)/x^2$, and $g(x) = 1+\cos x$. Then our function is $I \sim f(x)g(\xi x)$, where $x := \pi(a/\lambda)\sin\theta$, and $\xi x := 2\pi(d/\lambda)\sin\theta$, with $\xi \gg 1$ since $d \gg a$.

Using a computer programme, plot the three curves $y = f(x)g(\xi x)$, $y = 2f(x)$, and $y = g(\xi x)$ for particular values of ξ (e.g. $\xi = 4$). It is particularly useful if you manage to plot them in the same figure. Use some programme to find the approximate position of the maxima (in class we have determined their position approximatively; and we have determined exactly the position of the minima).

In case you cannot do it, on the next page you can see my plots, for $\xi = 4$. For convenience, on the x axis I am actually indicating x/π, which is easier to visualise (so when you read, for instance, ± 1 on the x axis, it corresponds to $x = \pm\pi$).

The blue plot corresponds to $y = f(x)g(4x)$ (interference + diffraction), the green one is the diffraction from a single slit which I multiplied by two to normalise it to the same total intensity of the previous curve – hence it is $y = 2f(x)$. Finally, the purple plot is the interference between the two slits discarding diffraction from single slits, namely $y = g(4x)$.

Note that the maxima of the blue curve are approximatively where those of the purple curve are. Note also that the position of the minima of the curves are exactly the same. Finally, note that the minima from the diffraction from single slit curve (green curve) are much farther apart – their position is a multiple of λ/a which is much larger than λ/d.

Finally, try to see what happens when the frequency of oscillations become very large, or, equivalently, $\lambda \to 0$ (you can arrange this by replacing x by $x\times$ a very large number; try for example $x \to 30x$. This is the classical limit...what happens to the diffraction figure? Have fun!

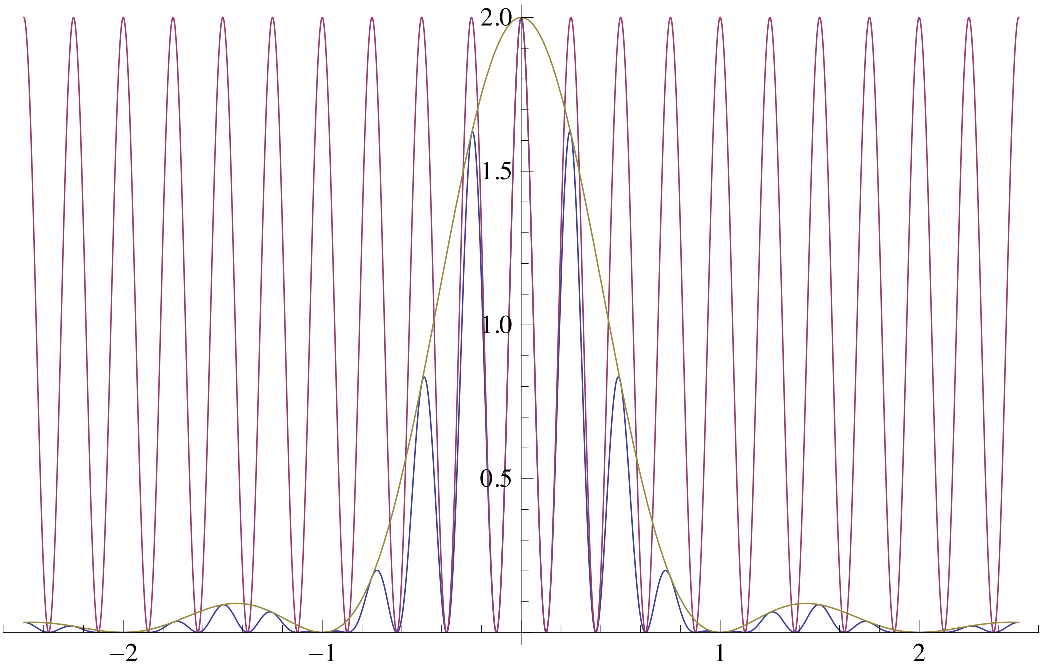

Figure 1: *Blue: diffraction + interference from the two slits. Purple: interference from two slits discarding diffraction from single slits. Green: diffraction from a single slit.*

Useful constants: Planck constant $h = 6.62 \times 10^{-34}$ J s. Boltzmann constant $k = 1.38 \times 10^{-23}$J/K. Speed of light $c = 3 \times 10^{8}$m/s. Electron mass: $m_e \simeq 0.5$ Mev/c^2. $\hbar c \sim 197$ MeV $\times$ fermi.