Quantum Physics – Homework 2

Due Thursday 26th January 2012 at 4. Attempt answers to all questions.

Hand in your script by the deadline into the post box near the secretaries' offices on level 1. **Assignments handed in past the deadline will not be marked.** Course title, week number and student name should appear on every sheet of the worked exercises, which should be securely bound together. Please *also* write your tutor's initials and time of tutorial on the cover sheet.

Problem 1 `[18 marks]`

Consider the following three bodies: (a) A red star, whose surface temperature is 3000 K; (b) a blue star, whose surface temperature is 10000 K; (c) the human body, at a temperature of 37°C. Assuming that these can all be treated as a blackbody, determine for each case:

(i) the total emissive power R (from Stefan-Boltzmann law); **[9]**

(ii) The wavelength $\lambda_{\max}$ where the spectral emittance $R(\lambda, T)$ has a maximum. **[9]**

Note: remember to convert Celsius degrees in Kelvin! You will also need Stefan's constant $\sigma = 5.67 \times 10^{-8}\mathrm{Wm^{-2}K^{-4}}$ and Wien's constant $b = 2.898 \times 10^{-3}$ m K.

Problem 2 `[12 marks]`

Using Planck's formula $E = h\nu$, determine the energy expressed in eV of photons whose wavelentgth is equal to (a) 10^{-13}m (γ rays); (b) 5×10^{-7} m (visible light); (c) 10 m (radio waves). **[12]**

Note: first thing to do is to convert wavelentghs into frequencies!

Problem 3 `[20 marks]`

Planck's formula for the spectral emittance of a blackbody is

$$R(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} ,$$

where T is the temperature, λ is the wavelentgh of the radiation emitted.

(i) Derive an approximated formula for $R(\lambda, T)$ valid for large λ (or, equivalently, for large temperature T) by performing an appropriate Taylor-expansion of Planck's formula for $R(\lambda, T)$. This approximate expression is called *"Rayleigh-Jeans"* formula. **[12]**

(ii) Explain why the Rayleigh-Jeans formula derived in part (i) leads to the ultraviolet catastrophe, and... **[4]**

(iii) ...explain why Planck's formula does not lead to the ultraviolet catastrophe! **[4]**

Useful constants: Planck constant $h = 6.62 \times 10^{-34}$ J s. Boltzmann constant $k = 1.38 \times 10^{-23} J/K$. Speed of light $c = 3 \times 10^8$m/s.