Quantum Physics – Homework 1

Due Thursday 19th January 2012 at 4. Attempt answers to all questions.

Hand in your script by the deadline into the post box near the secretaries' offices on level 1. **Assignments handed in past the deadline will not be marked.** Course title, week number and student name should appear on every sheet of the worked exercises, which should be securely bound together. Please *also* put your tutor's initials and time of tutorial on the cover sheet.

Problem 1 `[40 marks]`

Consider a particle of mass $m = 1$ Kg which can move along the x-axis and for $x > 0$. It is subject to a conservative force, whose potential is

$$V(x) \;=\; -V_0 \left(\frac{a^2}{x^2} - \frac{a^4}{x^4} \right) \;,$$

where V_0 and a are constants: $V_0 = 2\,\mathrm{J}$ and $a = 1\,\mathrm{m}$.

(i) What is the equilibrium position x_0 for the particle in this potential? **[6]**

(ii) Draw a sketch of the potential for $x > 0$. **[6]**

(iii) Use the harmonic oscillator approximation to describe the motion of this particle around the equilibrium position. What is the frequency of the oscillatory motion? **[8]**

(iv) If the maximum displacement from the equilibrium position is $A = 1$cm, what is the total (non-relativistic) energy stored in the particle? Is it constant? (Note: here we write the solution as $x(t) = x_0 + A\cos(\omega t + \phi)$, where ω is the frequency of the oscillatory motion). **[8]**

(v) What is the average kinetic energy over one period of oscillation? **[6]**

(vi) What is the average potential energy over one period of oscillation? **[6]**

Suggestion: Please first work out the various quantity which I asked you to compute without plugging the explicit numerical values indicated – plug these only after you have completed the calculation in terms of the various parameters!

Problem 2 `[10 marks]`

An electron has a mass equal to (approximatively) $m \simeq 9.2 \times 10^{-31}$ kg.

(i) Calculate its rest energy mc^2 and express it in MeV $= 10^6$ eV (recall that 1 eV $= 1.6$ 10^{-19} J. **[2]**

Using the relativistic expression for the energy of the electron $E = mc^2\gamma$, find the kinetic energy $T = E - mc^2$ expressed in eV, in the case where the speed of the electron is:

(ii) $v = 10^{-4}\, c$; **[2]**

(iii) $v = 10^{-2}\, c$; **[2]**

(iv) $0.3\, c$; **[2]**

(v) $0.999\, c$. **[2]**

Here c is the speed of light, $c \simeq 3 \times 10^8$ m/s.