Question paper 2, sample answers

Q1.
Solar Constant, S = 1373 W m^{-2} @ 1 A.U. (I will accept anything from 1350-1380!!)

It falls off as R^2 from the sun i.e. $S_{mars} = S_{earth} \times R^2_{earth}/R^2_{Mars} = 1373/(1.53)^2$

$= 586.5$ W m^{-2}

$R_{min} = 1.53\ (1-0.093) = 1.53 \times 0.907 = 1.388$

$R_{max} = 1.53\ (1 + 0.093) = 1.53 \times 1.093 = 1.672$

$S_{min} = 1373/1.388^2 = 712.97$ W m^{-2}

$S_{max} = 1373/1.672^2 = 490.96$

Difference between S_{mi}n and S_{max} = 222.01 W m^{-2}
(45% increase from farthest to nearest point)

Radiation balance means radiation energy from Sun = radiation in i.r from planet

So $(1\text{-}A)\ \pi\ r^2_{mars}\ S_{mars} = 4\pi\ r^2_{mar}s\ \sigma T^2_{eff}$

Where A is albedo, Smars is solar constant at Mars, r_{mars} is the radius of the planet, T_{eff} is the effective temperature and σ is Stefan=Boltzmann constant.

So $(1\text{-}A)\ S_{mars} = 4\ \sigma\ T^2_{eff}$

$T_{eff} = \{(1\text{-}0.15) \times 586.5/(4 \times 5.67\ 10^{-8})\}^{1/4} = 216.5$ K

The surface is actually warmer than this because of the greenhouse effect due to CO_2 in the atmosphere.

Mechanism: In-coming radiation is partly absorbed - (1-A) – by the surface and then re-radiated back up in the infra-red. Some of the radiation is then trapped by the CO_2 in the atmosphere, re-radiated, trapped, re-radiated etc until it reaches high enough in the atmosphere to be radiated to space. It is radiated to space with a local "temperature" which will be less than the ground. (Another way of puttin this is that every time it is absorbed and re-radiated it leaves some of its thermal energy in the atmosphere, warming the gas).

An alternative way of looking at this crudely is to compare a crude greenhouse to an open atmosphere"

Open atmosphere Energy in = E_{in}, amount that leaves is $E_{out} = (1\text{-}A)\ E_{in}$, temperature of ground T_0

With greenhouse roof (assume single layer letting visible light in but absorbing i.r.)
The E_{in} becomes $E_{out} = (1\text{-}A)\ E_{in}$ as it is reflected from the ground but this is then stopped by the "roof" which absorbs it and then re-radiates it half up and half down. If T_g is the temperature of the "roof" then $2\ \sigma T_g^4 = \sigma T_0^4$ i.e $T_g = (1/2^{1/4})\ T_0$

Of course not all i.r. is absorbed by the "greenhouse" on Mars and there is not just one absorbing and emitting layer, but this gives the principle.

Water on the surface. See phase diagram for water described in lecture notes, and note Clausius-Clapeyron equation. Triple point is 273.16 K so under nearly all circumstances water in the surface would be below the triple point. This means the water would freeze to ice, but the ice would be in the part of the phase diagram where it is in equilibrium with water vapour at a lower vapour pressure than liquid water. This any gas given off finds it easier to turn directly to gas than to have to go via liquid water. Thus the ice "sublimes" – the low atmospheric pressure and low water vapour content would mean that the ice would gradually sublime away to nothing, not ever giving a liquid component. A drop of water (liquid) dropped onto the surface may actually last for a period as liquid before turning to ice, even at temperatures much below the freezing point because water needs a nucleus or "trigger" to make it start to crystallise. Thus you may get super-cooled liquid on the surface for a short time, but this would not last long before it turned to ice.

Q2.

Volume of cloud $= \pi\ 2.5^2 \times 5 = 98.17\ km^3 = 98.17\ 10^9\ m^3$

So number of raindrops = $98.17\ 10^{17}$ drops

Volume of each drop on average = $4/3\ \pi\ (10^{-6})^3\ m^3 = 4.18\ 10^{-18}$ m3 per drop

Therefore volume of water in cloud = $41\ m^3$

Mass of water in cloud = 41×10^3 kg

Energy released = $41 \times 10^3 \times 2.5\ 10^6\ J = 102.6\ 10^9\ J = 1.03\ 10^{11}\ J$

For the air in the cloud $P=P_0\ e^{-z/H}$ where H is the scale height

Perfect Gas Law gives $P = \rho R_s T$, so for constant T $\rho = \rho_0\ e^{-z/H}$

Mass of a "sliver" of air dz high $= \pi R^2 \rho(z)\ dz$ where R is the radius of the column of cloud

So mass of air in cloud = Integral from 1km to 6 km of $\pi R^2 \rho(z)\ dz$

$= \pi\ (2.5\ 10^3)^2\ \rho_0$ (Integral from 1 to 6) $e^{-z/H}$

$= \pi\ (2.5^2\ 10^6) \times 1.24 \times 8.5\ 10^3\ (e^{-1/8.5} - e^{-6/8.5}) = 8.195\ 10^{10}$ kg

Kinetic energy $= ½\ mv^2 = ½\ 8.195\ 10^{10}\ (15)^2 = 9.222\ 10^{12}$ J

(You might also add the K.E. of the water = $½ \times 41 \times 10^3 \times 15^2$ - obviously a lot smaller)

Heat rise = Energy/(mass x Cp) = $1.03\ 10^{11}/(8.195\ 10^{10} \times 1000) = 1.25\ 10^{-3}$ K

Energy discharged in lightning = qV = 20 x 100,000 J = $2\ 10^6$ J

Assume at cloud base we have 10^5 V and distance from the ground = 1 km = 10^3 m,

so P.D. = $10^5/1000 = 10^2$ V/m

Surface charge density $\sigma = \varepsilon_0 E = 8.85\ 10^{-12}\ 10^2 = 8.85\ 10^{-10}\ C/m^2$

Assume this was of the order of charge released in the lightning – i.e. 20C

Then depth of charge layer dz is given by

$\pi\ (2.5 \times 10^3)^2\ dz = 20/(8.85 \times 10^{-10})$ so dz = 1150 m

In 1150 m there are $\pi\ (2.5 \times 10^3)^2 \times 1150 \times 10^8$ drops = $22.5\ 10^{17}$ drops

20C -> $20/1.6\ 10^{-19}$ charges = $12.5\ 10^{19}$ charges

Therefore the average charge per drop = $12.5\ 10^{19}/22.5\ 10^{17}$ = 56 charges per water drop

[Note: this line of argument is very questionable. You will get extra marks if you can tell me the logical flaw in this argument!! ADA]

Q3.

Water potential is the effective gravitational potential height of the water in the soil, after taking into account all the sources of pressure on that water such as the “suction” due to surface tension plus osmotic pressure. Suction is expressed in cm of water to give it the same form as the gravitational potential, which is given as cm of water column (pressure = height of column x g). As in all systems the “pressure” as far as movement is concerned is to make the water move to where it reduces potential. Since the depth goes up as we go down but obviously the potential should be less, the depth should have a minus sign when we go down. Thus if we go from 10 cm depth to 20 cm depth, we have changed potential by g(-20 –(-10)) = -10g = so the water would have lost potential, and “down” is its natural direction in which it is pulled (as it is for any object in a gravitational field. However, with the suction if we look at the difference between the top two layers – change in potential going from 10 to 20 cm depth is (-120 –(-300)) = +180 – ie it would be moving against the suction force which is trying to pull it up.

This is expressed mathematically as water potential Ψ given by

$\Psi = \Psi d + \Psi s$ where Ψd is the depth and Ψs is the suction

And then potential difference between two places is given by $d\Psi/dz$

Since z is negative for depth we use the values in the table such that the force is up if $d\Psi/dz$ is positive and down if negative.

Thus, between 10 and 20cm depth: $d\Psi/dz = ([120+20]-[300+10])/(-20+10) = 17$

As this is positive the force is upwards - that is the soil near the surface is so dry that the suction forces in the soil are trying to pull water upwards from the reservoir below

What of 50-60 cm? Well we only have values at 40 and 50 cm so to find the potential gradient between 50 and 60 we need to extrapolate downwards. The depth at 60 cm isobviosly 60 but we do not know the “suction”. We can use a variety of methods of different sophistication to find the value os “suction” at 60cm but they will all obviously give something around 79-80 if we look at the trend in the table. Thus we can say the potential difference is given by :

$d\Psi/dz = ([80+60]-[83+50])/(-60-50) = -0.7$

This is negative so the force on the water at these levels is attempting to pull the water downwards – the soil obviously has enough water in it that its own weight under gravity is the major redistributive force.

Q4

For derivation of Geostrophic balance see lecture notes or
http://www.apl.ucl.ac.uk/lectures/3c37/3c37-9.html

This leads to the equation for geostrophic balance $-(1/\rho)\, dP/dx = 2\, u\, \Omega \sin\varphi$

Where ρ is the atmospheric density, Ω is the rate of rotation of the earth about its axis and φ is the latitude. This can be expressed as $-(1/\rho)\, dP/dx = u\, f_c$ where $f_c = 2\, \Omega \sin\varphi$

If we substitute for these parameters, $\Omega = 2\pi$ radians/day $= 2\pi/(24 \times 3600)$ radians/s $= 7.27\ 10^{-5}$ rad/s

So $f_c = 1.45\ 10^{-4} \sin\varphi$ radians/s

```
Geostrophic balance        ^  dP/dx
                            |
                            |-------------> velocity
                            |
                           v  Coriolis
                               Force            (N.hemisphere)

Cyclostrophic Balance
                            ^  dP/dx
                             |
                             |-----------> velocity
                             |
                            v  v^2/r centrigugal force
```

in the case of the Geostrophic balance, the $\sin\varphi$ term changes sign between northern and southern hemispheres and so the direction of velocity which balances the pressure term changes direction. Thus cyclones and anticyclones reverse direction between the hemispheres.

The Gradient wind equation combines the two forces, geostrophic and cyclostrophic to give the combined force which balances the pressure gradient:

$$V^2/R + f_c V = (1/\rho)\ dP/dx$$

Now for the two different places in the hurricane we can substitute into these different terms and see which one dominates

So if location A is the inner one, wind (u) = 50 m/s and R = 25km, and location B is the outer one where velocity = 10 m/s and R = 400km, ansd we have $f_c = 1.45\ 10^{-4}\ \sin(30) = 0.725\ 10^{-4}$ rads/s

Location	V^2/R	$f_c V$
A	0.1	0.0036
B	$2.5\ 10^{-4}$	$7.25\ 10^{-4}$

So we see that near the eye of the storm the wind is largely cyclostrophic while near the outside it is more geostrophic, though there is still a large cyclostrophic component.