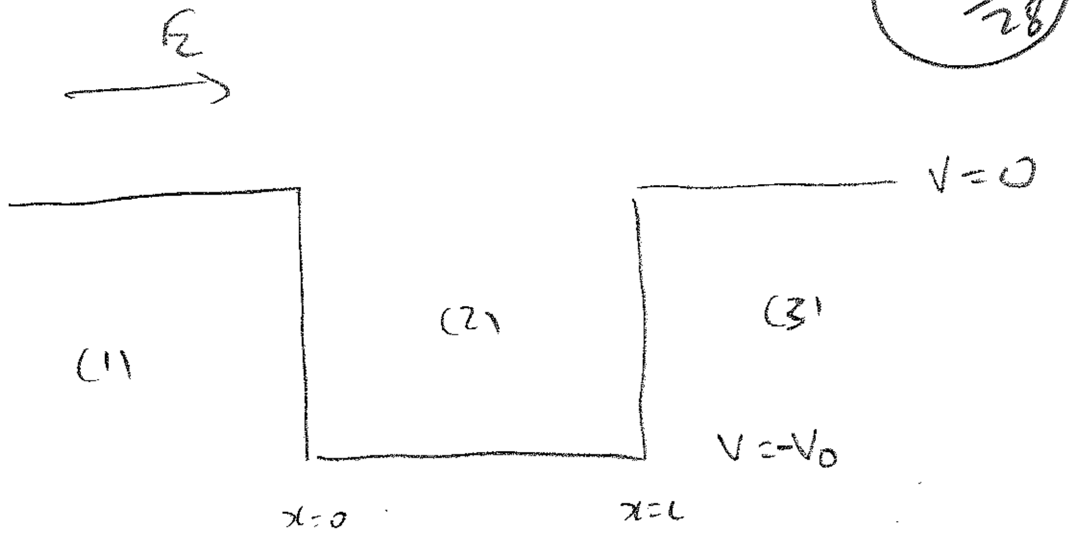
QMA SOLUTIONS 6A

TOTAL $\frac{}{28}$



REGIONS (1) & (3) $V = 0$

TISE ⇒ $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + 0 = E\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

SOLUTIONS

(1) $$\psi = \underset{\text{INCIDENT}}{Ae^{ikx}} + \underset{\text{REFLECTED}}{Be^{-ikx}}$$ ✓ ✓

(3) $$\psi = \underset{\text{TRANSMITTED}}{Fe^{ikx}} + \overset{=0}{Ge^{-ikx}}$$ ✓

REGION (2) $V = -V_0$

TISE ⇒ $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi \qquad q = \sqrt{\frac{2m(V_0+E)}{\hbar^2}}$$

$$\frac{d^2\psi}{dx^2} = -\frac{2m(E+V_0)}{\hbar^2}\psi = -q^2$$

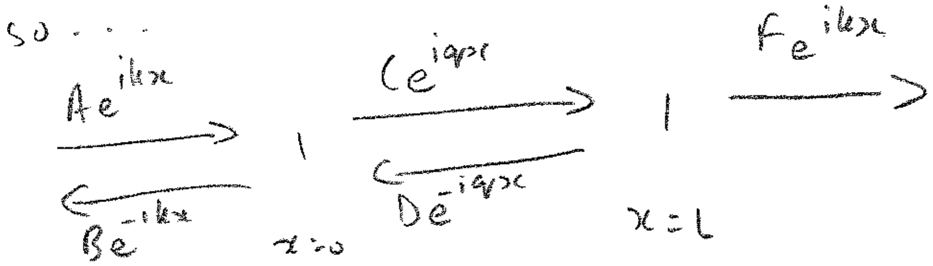
SOLUTIONS $\psi = Ce^{iqx} + De^{-iqx}$ ✓✓

RIGHT TRAVELLING (TRANSMITTED AT $x=0$)

LEFT TRAVELLING (REFLECTED AT $x=L$)

$/5$

SO . . .



AT $x=0$ AND $x=L$ ψ CONTINUOUS ✓✓

AND $\frac{d\psi}{dx}$ CONTINUOUS

(THE WAVEFUNCTION IS CONTINUOUS AND SMOOTH)

AT $x=0$

ψ CONTINUOUS $Ae^{ikx} + Be^{-ikx} = Ce^{iqx} + De^{-iqx}$

$\Rightarrow \quad A + B = C + D$ ① ✓

$\frac{d\psi}{dx}$ CONTINUOUS $ik(Ae^{ikx} - Be^{-ikx}) = iq(Ce^{iqx} - De^{-iqx})$

$\Rightarrow \quad k(A-B) = q(C-D)$ ② ✓

AT $x = L$

ψ CONTINUOUS $$Ce^{iqx} + De^{-iqx} = Fe^{ikx}$$

$$\Rightarrow \quad Ce^{iqL} + De^{-iqL} = Fe^{ikL} \qquad ③ \quad ✓$$

$\frac{d\psi}{dx}$ CONTINUOUS $$iq(Ce^{iqx} - De^{-iqx}) = ikFe^{ikx}$$

$$\Rightarrow \quad q(Ce^{iqL} - De^{-iqL}) = kFe^{ikL} \qquad ④ \quad ✓$$

NOW $$T = \left|\frac{j_{TRANS}}{j_{INCIDENT}}\right| = \left|\frac{F}{A}\right|^2 \quad ✓$$

SINCE FOR $\Psi_{(x,t)} = Ae^{ikx}e^{-\frac{iEt}{\hbar}}$

$$j = |A|^2\frac{\hbar k}{m}$$

& k, m SAME IN (1) & (5)

NEED $\frac{F}{A}$ . . .

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$k①+②$

$$\Rightarrow \quad 2kA = (q+k)C + (k-q)D \quad ⑤$$ ✓

$q③+④$

$$\Rightarrow \quad 2qCe^{iqL} = (k+q)Fe^{ikL} \quad \Rightarrow \quad C = \frac{1}{2q}(k+q)\frac{Fe^{ikL}}{e^{iqL}}$$ ✓

$$= \frac{1}{2q}(k+q)Fe^{ikL}e^{-iqL}$$

$q③-④$

$$\Rightarrow \quad 2qDe^{-iqL} = (q-k)Fe^{ikL}$$

✓

$$\Rightarrow \quad D = -\frac{1}{2q}(k-q)Fe^{ikL}e^{+iqL}$$

SUBSTITUTE INTO ⑤

✓

$$2kA = (k+q)\cdot\frac{1}{2q}(k+q)Fe^{ikL}e^{-iqL} + (k-q)\left(-\frac{1}{2q}\right)Fe^{ikL}e^{iqL}(k-q)$$

$$4qk\,A = (k+q)^2Fe^{ikL}e^{-iqL} - (k-q)^2Fe^{ikL}e^{iqL}$$

✓

$$\frac{A}{F} = \frac{(k+q)^2e^{ikL}e^{-iqL} - (k-q)^2e^{ikL}e^{iqL}}{4qk}$$

$$\frac{F}{A} = \frac{4qk}{(k+q)^2e^{ikL}e^{-iqL} - (k-q)^2e^{ikL}e^{iqL}}$$ ✓

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$$T = \left|\frac{F}{A}\right|^2 = \left(\frac{F}{A}\right)^* \left(\frac{F}{A}\right)$$ ✓

$$T = \left(\frac{4qk}{(k+q)^2 e^{-ikL} e^{+iqL} - (k-q)^2 e^{-ikL} e^{-iqL}}\right)\left(\frac{4qk}{(k+q)^2 e^{ikL} e^{-iqL} - (k-q)^2 e^{ikL} e^{iqL}}\right)$$ ✓

$$= \frac{16q^2k^2}{\left((k+q)^2 e^{-ikL} e^{+iqL} - (k-q)^2 e^{-ikL} e^{-iqL}\right)\left((k+q)^2 e^{ikL} e^{-iqL} - (k-q)^2 e^{ikL} e^{iqL}\right)}$$

$$= \frac{16q^2k^2}{\left((k+q)^2\right)^2 - (k+q)^2(k-q)^2 e^{2iqL} - (k-q)^2(k+q)^2 e^{-2iqL} + \left((k-q)^2\right)^2}$$ ✓

$$T = \frac{16q^2k^2}{\left((k+q)^2\right)^2 + \left((k-q)^2\right)^2 - (k+q)^2(k-q)^2\left(e^{2iqL} + e^{-2iqL}\right)}$$

Now $a^2+b^2 = (a-b)^2 + 2ab$ ✓

$$\left((k+q)^2\right)^2 + \left((k-q)^2\right)^2 = \left((k+q)^2 - (k-q)^2\right)^2 + 2(k+q)^2(k-q)^2$$

And $e^{2iqL} + e^{-2iqL} = \left(e^{iqL} - e^{-iqL}\right)^2 + 2$ ✓

$$T = \frac{16q^2k^2}{\left\{(k+q)^2 - (k-q)^2\right\}^2 + 2(k+q)^2(k-q)^2 - (k+q)^2(k-q)^2\left\{\left(e^{iqL} - e^{-iqL}\right)^2 + 2\right\}}$$

$$= \frac{16q^2k^2}{\left((k+q)^2 - (k-q)^2\right)^2 - (k+q)^2(k-q)^2\left(e^{iqL} - e^{-iqL}\right)^2}$$

✓

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

$$T = \frac{16q^2k^2}{\left((k+q)^2 - (k-q)^2\right)^2 - (k+q)^2(k-q)^2 \cdot (-4)\sin^2(qL)}$$

$$T = \frac{16\,q^2k^2}{\left((k+q)^2 - (k-q)^2\right)^2 + 4(k+q)^2(k-q)^2 \sin^2(qL)}$$

NOW $(k+q)^2(k-q)^2 = k^4 - 2k^3q + k^2q^2 + 2k^3q - 4k^2q^2 + 2kq^3$
$+ q^2k^2 - 2kq^3 + q^4$ ✓

$= k^4 - 2k^2q^2 + q^4$ ✓

$= (k^2 - q^2)^2$

AND $\left((k+q)^2 - (k-q)^2\right)^2 = \left((k^2 + 2qk + q^2) - (k^2 - 2qk + q^2)\right)^2$

$= (4qk)^2 = 16q^2k^2$ ✓

$\therefore$

$$T = \frac{16q^2k^2}{16q^2k^2 + 4(k^2-q^2)^2\sin^2(qL)}$$ /10

✓

$$T = \frac{4q^2k^2}{4q^2k^2 + (k^2-q^2)^2\sin^2(qL)}$$ AS REQUIRED

TOTAL /28