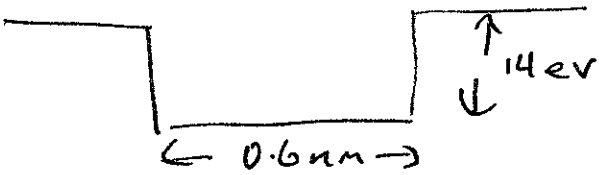
QMA PROBLEMS 3A SOLUTIONS

TOTAL /25

1) $\frac{L}{2} = 0.6\text{nm}$ $\quad V_0 = 14\text{eV}$



a) $\xi_0 = \frac{L}{2}\sqrt{\frac{2mV_0}{\hbar^2}} = 3\times10^{-10}\sqrt{\frac{2\times 5.11\times10^5\times 14}{9\times10^{16}\times(6.58\times10^{-16})^2}}$

$\xi_0 = 5.75$ ✓

~~NOT~~ $\frac{2\xi_0}{\pi} = 3.65$ ✓ 1 STATE FOR $\frac{\pi}{2}$ INTERVAL SOME ARGUMENT

$\therefore$ 4 BOUND STATES ✓ /3

b) $n_1 = 1.34$

$n_2 = 2.66$

$n_3 = 3.95$

$n_4 = 5.17$

$n = \frac{kL}{2} = \frac{L}{2}\sqrt{\frac{2mE}{\hbar^2}}$

$\frac{4n_n^2\hbar^2}{2mL^2} = E_n$

$E_n = n_n^2 \cdot \frac{4\hbar^2}{2mL^2}$ ✓

$E_1 = 1.34^2 \times 0.424 = 0.761\text{eV}$ ✓

$E_2 = 2.66^2 \times 0.424 = 3.00\text{ eV}$ ✓

$E_3 = 3.95^2 \times 0.424 = 6.62\text{ eV}$ ✓

$E_4 = 5.17^2 \times 0.424 = 11.33\text{eV}$ ✓

$\frac{4\hbar^2}{2mL^2} = \frac{4\times(6.58\times10^{-16})^2\times 9\times10^{16}}{2\times 5.11\times10^5\times(6\times10^{-10})^2}$

$= 0.424\text{ eV}$ ✓

/6

A10

c) $\frac{hc}{\lambda} = \Delta E = E_4 - E_3$ ✓ $\quad \lambda = \frac{hc}{\Delta E}$

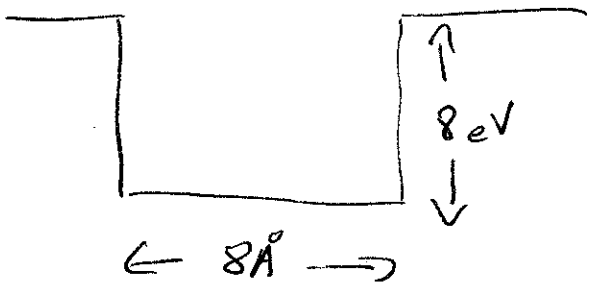
$\lambda = \frac{4.13\times10^{-15} \times 3\times10^{8}}{(11.33 - 6.62)} = 2.63 \times 10^{-7}\,m$ ✓

$\lambda = 263\,nm$ (U.V. & CLOSE TO EXPECTATION!)

(BONUS IF CORRECT)

2/2

2)



d) AS $V_0 \to \infty$ $z_0 \to \infty$

$(n-1)\frac{\pi}{2} < \eta_n \leq n\frac{\pi}{2}$ so $\eta_n \to n\frac{\pi}{2}$ ✓

$E_n = \eta_n^2 \frac{4\hbar^2}{2mL^2}$, $E_n \to \frac{n^2\pi^2}{4}\frac{4\hbar^2}{2mL^2}$ ✓

$E_n \to \frac{n^2\hbar^2\pi^2}{2mL^2}$

3/3

a) $z_0 = \frac{L}{2}\sqrt{\frac{2mV_0}{\hbar^2}} = 4\times10^{-10}\sqrt{\frac{2\times 5.11\times10^{5} \times 8}{1\times10^{16}\,(6.58\times10^{-16})^2}}$

$z_0 = 5.79$ ✓

✓ 1 STATE PER $\frac{\pi}{2}$ INTERVAL SOME REASONING

$\frac{2z_0}{\pi} = 3.68 \quad \therefore \quad 4$ BOUND STATES ✓

3/3

b) See attached graphs on sin, findroot ...

SO... 1 x 4 SOLUTIONS

BUT ONLY IF JUSTIFIED

+4

8/8

NUMBERS ONLY → 2/8

In[1]:= **FindRoot[Tan[t] == t, t]**

FindRoot::fdss : Search specification t should be a list with 1 to 5 elements. >>

Out[1]= FindRoot[Tan[t] == t, t]

In[3]:= **FindRoot[Tan[t] == t, {t, 1}]**

Out[3]= $\{t \to 1.48307 \times 10^{-8}\}$

In[4]:= **z = 5.79**

Out[4]= 5.79

In[5]:= **FindRoot[Tan[t] == ((z / t) ^ 2 - 1) ^ 0.5, {t, 1}]**

Out[5]= {t → 1.33766}

In[6]:= **FindRoot[Tan[t] == ((z / t) ^ 2 - 1) ^ 0.5, {t, 2}]**

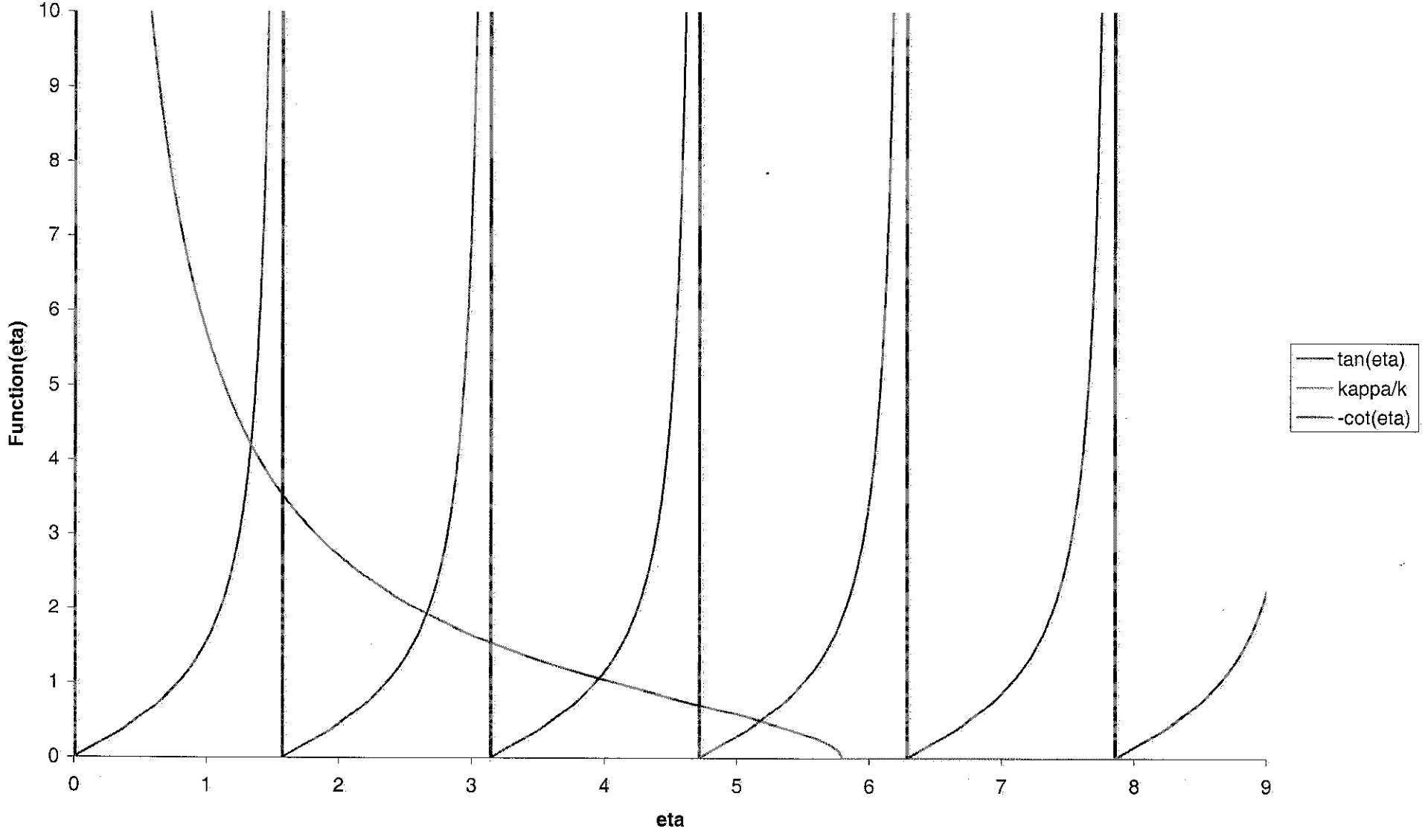
Out[6]= {t → 3.95939}

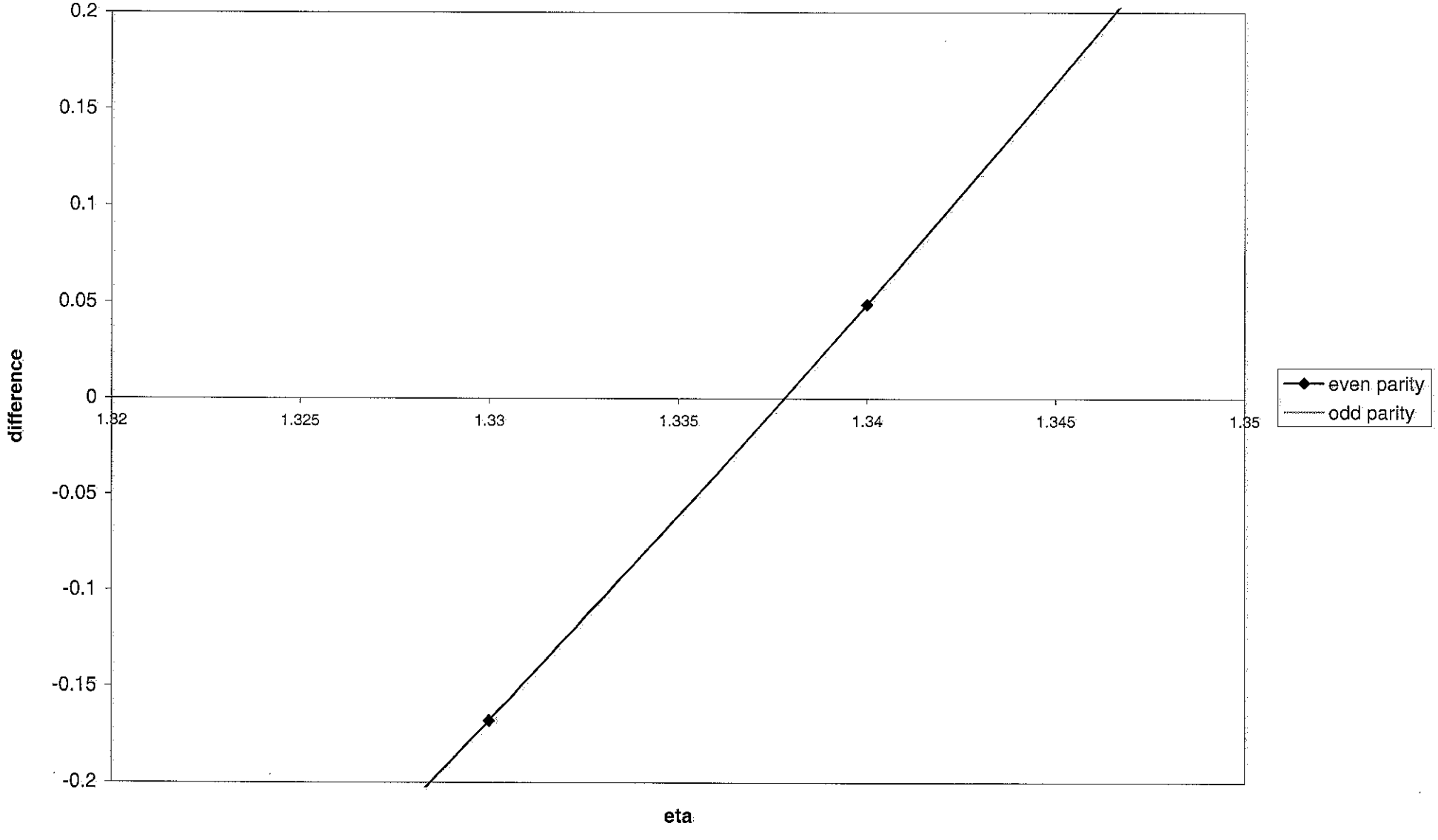
In[10]:= **FindRoot[-1 * Cot[t] == ((z / t) ^ 2 - 1) ^ 0.5, {t, 1}]**

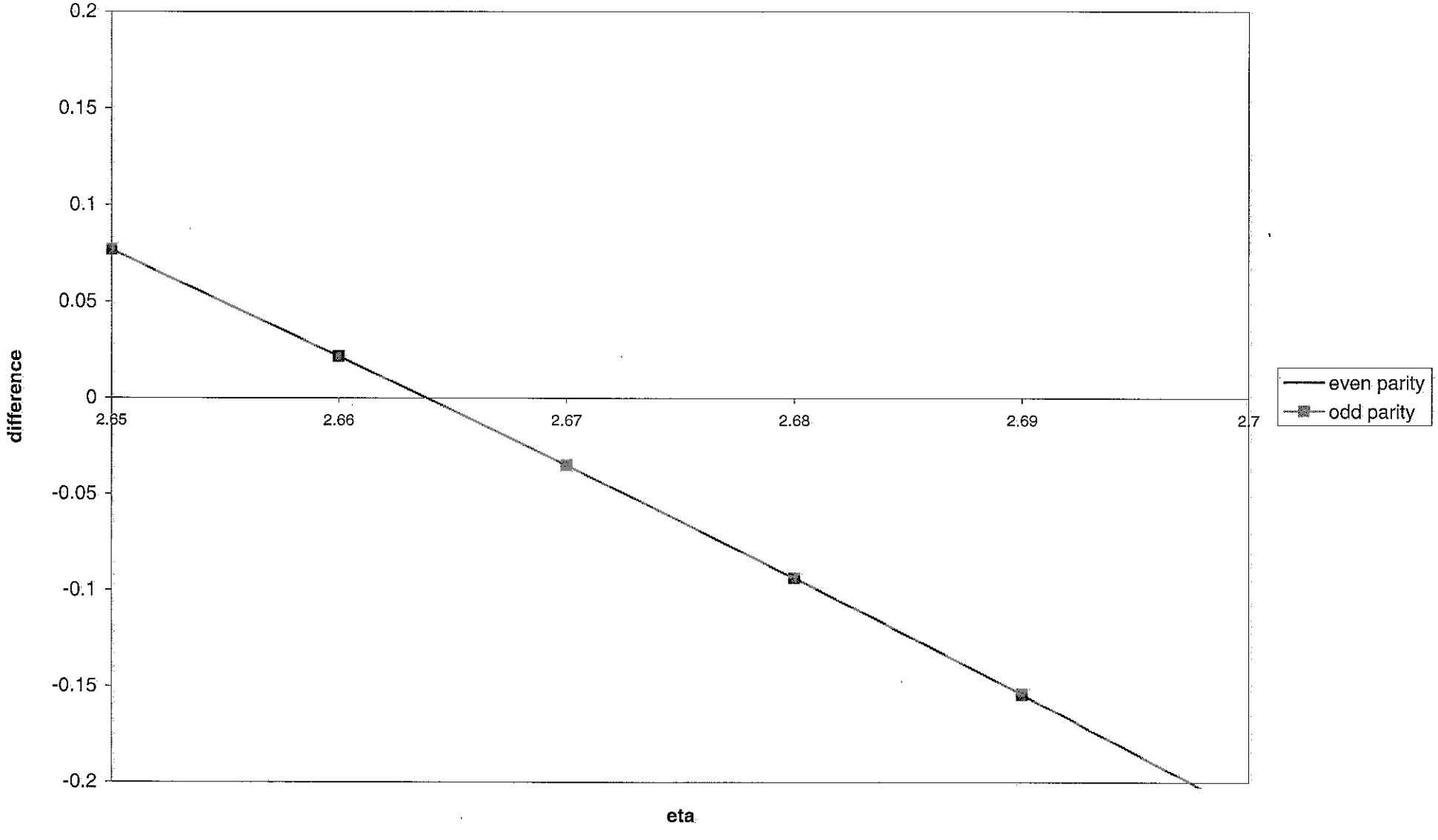
Out[10]= {t → 2.66357}

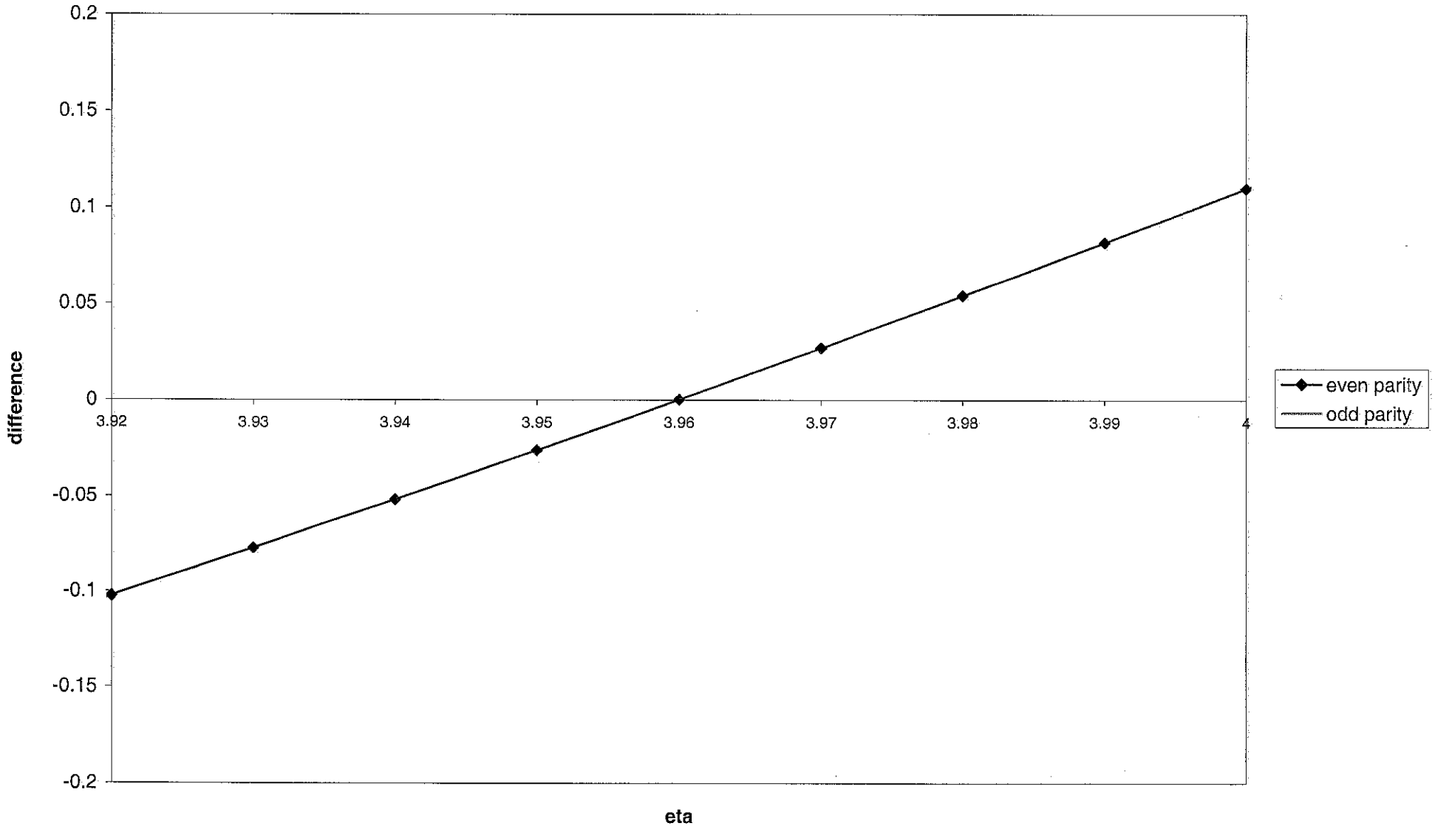
In[12]:= **FindRoot[-1 * Cot[t] == ((z / t) ^ 2 - 1) ^ 0.5, {t, 4}]**

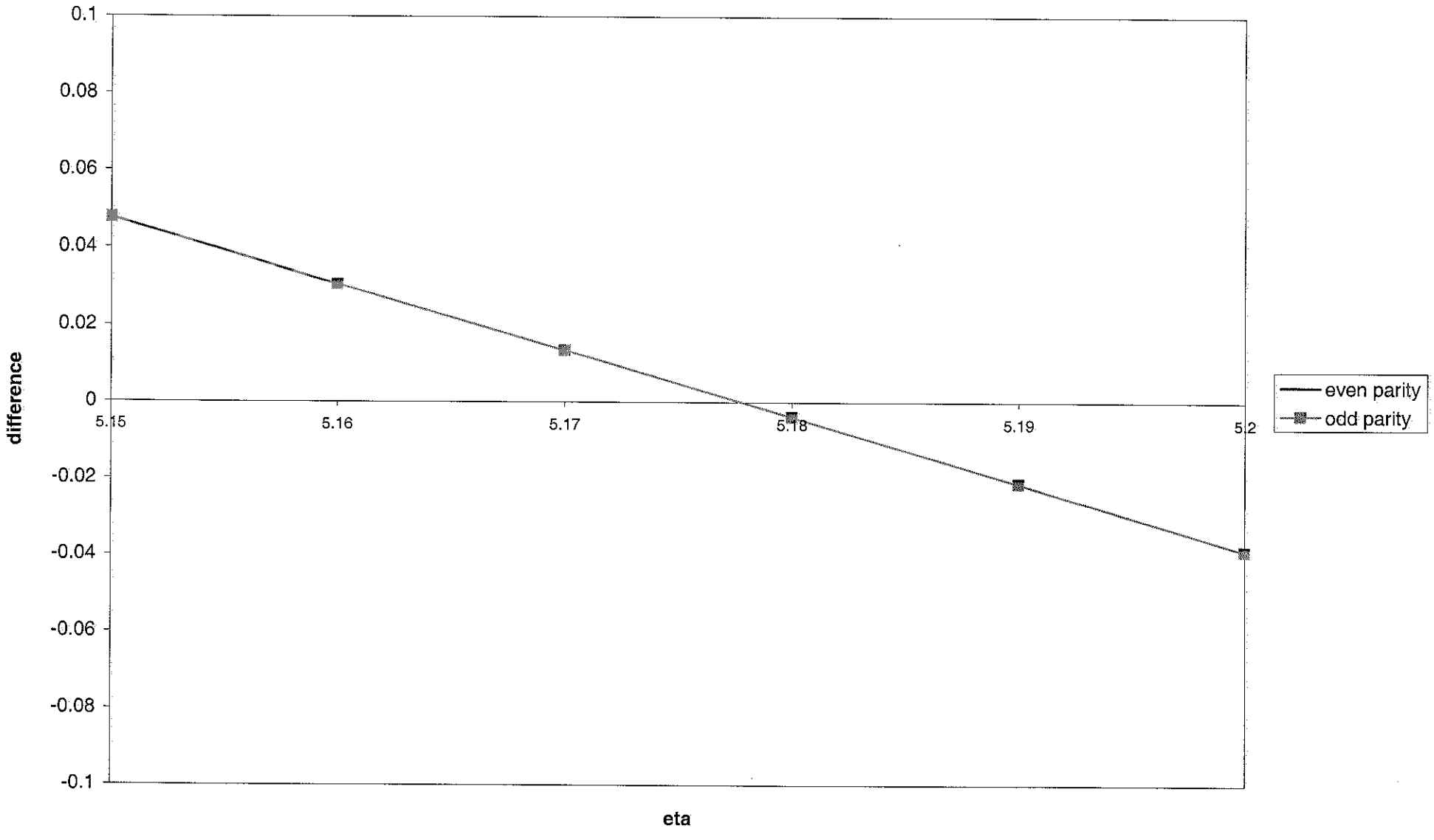
Out[12]= {t → 5.17679}


Function(eta)
eta
0
1
2
3
4
5
6
7
8
9
10
tan(eta)
kappa/k
-cot(eta)


0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
difference
1.32
1.325
1.33
1.335
1.34
1.345
1.35
eta
even parity
odd parity


0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
difference
2.65
2.66
2.67
2.68
2.69
2.7
eta
even parity
odd parity


0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
difference
3.92
3.93
3.94
3.95
3.96
3.97
3.98
3.99
4
eta
even parity
odd parity


0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
difference
5.15
5.16
5.17
5.18
5.19
5.2
eta
even parity
odd parity