QMA PROBS 2A SOLUTIONS

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A3

1) $$\Psi_n(x,t) = \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right) e^{-\frac{iE_n t}{\hbar}} \qquad -\frac{L}{2} \le x \le \frac{L}{2}$$

$n = 1, 3, 5 \ldots$

a) $$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \left(\sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right) e^{-\frac{iE_n t}{\hbar}}\right)}{\partial x^2} + 0 = i\hbar \frac{\partial \left(\sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right) e^{-\frac{iE_n t}{\hbar}}\right)}{\partial t}$$

$$-\frac{\hbar^2}{2m} \cdot -\frac{n^2\pi^2}{L^2} \cdot \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right) e^{-\frac{iE_n t}{\hbar}} = i\hbar \cdot -\frac{iE_n}{\hbar} \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right) e^{-\frac{iE_n t}{\hbar}}$$

$$+\frac{\hbar^2 n^2 \pi^2}{2mL^2} = +E_n$$

$$\therefore \quad E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

/5

b) $$dP = \Psi^* \Psi \, dx$$

$$_0P_{\frac{L}{4}} = \int_0^{\frac{L}{4}} \Psi^* \Psi \, dx = \frac{2}{L}\int_0^{\frac{L}{4}} \cos\frac{n\pi x}{L} e^{+\frac{iE_n t}{\hbar}} \cos\frac{n\pi x}{L} e^{-\frac{iE_n t}{\hbar}} dx$$

$$= \frac{2}{L}\int_0^{\frac{L}{4}} \cos^2\frac{n\pi x}{L}\, dx = \frac{2}{L}\cdot\frac{1}{2}\int_0^{\frac{L}{4}} 1 + \cos\frac{2n\pi x}{L}\, dx$$

A4

b) CONTD

$$P_{0 \to \frac{L}{4}} = \frac{1}{L} \int_0^{\frac{L}{4}} 1 + \cos \frac{2n\pi x}{L} \, dx$$

$$= \frac{1}{L} \left[x + \frac{L}{2n\pi} \sin \frac{2n\pi x}{L} \right]_0^{L/4}$$

$$P = \frac{1}{L} \left(\frac{L}{4} - 0 + \frac{L}{2n\pi} \sin \frac{n\pi}{2} - \frac{L}{2n\pi} \overbrace{\sin(0)}^{=0} \right)$$

$$= \frac{1}{L} \left(\frac{L}{4} + \frac{L}{2n\pi} (-1)^{\frac{n-1}{2}} \right) \quad \text{SINCE } n \text{ IS ODD}$$

$$P_{0 \to \frac{L}{4}} = \left(\frac{1}{4} + \frac{(-1)^{\frac{n-1}{2}}}{2n\pi} \right)$$

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$$= \frac{1}{4} + \frac{1}{2\pi} \qquad n = 1$$

$$= \frac{1}{4} - \frac{1}{6\pi} \qquad n = 3$$

$$= \frac{1}{4} + \frac{1}{10\pi} \qquad n = 5 \quad \text{etc}$$

As $n \to \infty \quad \frac{1}{n} \to 0$

so $P_{0 \to \frac{L}{4}} \to \frac{1}{4}$ FOR n LARGE ...

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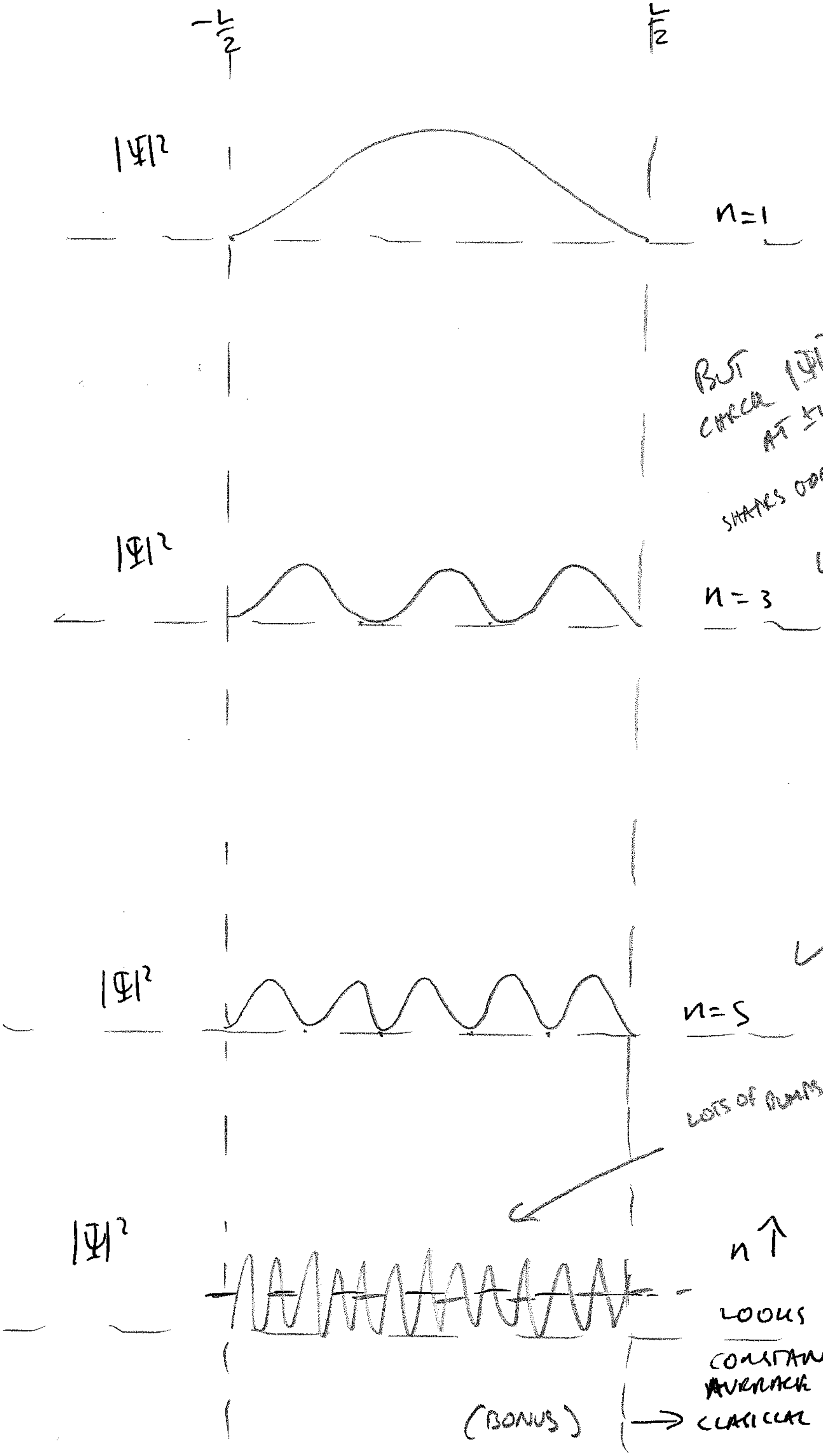
A5



BUT CHECK $|\Psi|^2$ is 0 AT $\pm\frac{L}{2}$

SHAPES GOOD, ALWAYS +VE

LOTS OF BUMPS

$\frac{}{4}$

LOOKS LIKE CONSTANT AVERAGE VALUE

(BONUS) → CLASSICAL LIMIT!

A6

1) f) $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$

$$\langle p \rangle = \frac{2}{L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} \cos\left(\frac{n\pi x}{L}\right) e^{+\frac{i E_n t}{\hbar}} \cdot (-i\hbar) \frac{\partial\left(\cos\left(\frac{n\pi x}{L}\right) e^{-\frac{i E_n t}{\hbar}}\right)}{\partial x} dx$$

$$= \frac{2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} \cos\left(\frac{n\pi x}{L}\right) (-i\hbar)\left(-\frac{n\pi}{L}\right) \sin\frac{n\pi x}{L} \, dx$$

$\langle p \rangle = 0$ ODD FUNCTION, INTEGRATED SYMMETRICALLY ABOUT ZERO

OR

$$\langle p \rangle = \frac{2}{L} i\hbar \frac{n\pi}{L}\left[\frac{L}{2n\pi} \sin^2 \frac{n\pi x}{L}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{2}{L} i\hbar n\pi \left(\overset{0}{\sin^2 \frac{n\pi}{2}} - \underset{0}{\sin^2 \frac{n\pi}{2}}\right)\frac{1}{2n}$$

$\langle p \rangle = 0$

AS EXPECTED, OF COURSE.

$$\langle p^2 \rangle = \frac{2}{L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} \cos\left(\frac{n\pi x}{L}\right) e^{+\frac{i E_n t}{\hbar}} (-\hbar^2) \frac{\partial^2\left(\cos\left(\frac{n\pi x}{L}\right) e^{-\frac{i E_n t}{\hbar}}\right)}{\partial x^2} dx$$

$$\langle p^2 \rangle = -\hbar^2 \frac{2}{L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} \cos\left(\frac{n\pi x}{L}\right) \cdot -\frac{n^2\pi^2}{L^2} \cos\left(\frac{n\pi x}{L}\right) dx$$

A7

$$\langle p^2 \rangle = \frac{2\hbar^2\pi^2 n^2}{L^3} \int_{-\frac{L}{2}}^{+\frac{L}{2}} \cos^2\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{\hbar^2\pi^2 n^2}{L^3} \int_{-\frac{L}{2}}^{\frac{L}{2}} 1 + \cos\left(\frac{2n\pi x}{L}\right) dx$$

$$= \frac{\hbar^2\pi^2 n^2}{L^3} \left[x + \frac{L}{2n\pi} \sin \frac{2n\pi x}{L} \right]_{-\frac{L}{2}}^{\frac{L}{2}}$$

n odd

$$= \frac{\hbar^2\pi^2 n^2}{L^3} \left\{ \frac{L}{2} + \frac{L}{2} + \frac{L}{2n\pi} \overset{0}{\sin(n\pi)} + \frac{L}{2n\pi} \underset{0}{\sin n\pi} \right\}$$

$$\langle p^2 \rangle = \frac{\hbar^2\pi^2 n^2}{L^3} \cdot L$$

$$\langle p^2 \rangle = \frac{\hbar^2\pi^2 n^2}{L^2} \quad \therefore \Delta p = \sqrt{\frac{\hbar^2\pi^2 n^2}{L^2} - 0}$$

$$\Delta p = \frac{\hbar\pi n}{L}$$ AS REQUIRED

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A8

Q2

a) $E_n = \frac{n^2 \hbar^2 \pi^2}{2mL^2} = n^2 \left(\frac{\hbar^2 \pi^2}{2mL^2}\right) = n^2 E_1$

$\hbar = 6.58 \times 10^{-16}$ eVs

$M_e = 0.511 \frac{MeV}{c^2}$

$L = 7.2 \times 10^{-10}$ m

$c = 3 \times 10^8$ ms^{-1}

$E_1 = \left(\frac{(6.58 \times 10^{-16})^2 \times \pi^2 \times 9 \times 10^{16}}{2 \times 5.11 \times 10^5 \times (7.2 \times 10^{-10})} \right)$

$E_1 = 0.726$ eV

$E_1 = 0.726$ eV
$E_2 = 2.904$ eV
$E_3 = 6.534$ eV
$E_4 = 11.616$ eV

+2

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b) $\frac{hc}{\lambda} = E_4 - E_3$

$h = 4.13 \times 10^{-15}$ eVs

$\lambda = \frac{hc}{(E_4 - E_3)} = \frac{4.13 \times 10^{-15} \times 3 \times 10^8}{11.62 - 6.53} = 2.44 \times 10^{-7}$ m

$= 244$ nm

(WHICH IS JUST 5% OUT FROM $\lambda_{measured}$ 258nm)

SENSIBLE COMMENT ARR OK

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