

QMA PROBS 1A SOLUTIONS

A1



$$\Psi(x,0) = N \quad -a \leq x \leq +a$$

$$= 0 \quad \text{ELSEWHERE}$$

- a) IT IS DOUBLE VALUED AT $\pm a$ ✓
 IT IS NOT SMOOTH AT $\pm a$ ✓ (THE 1ST & 2ND DERIVATIVES ARE UNDEFINED AT $\pm a$)

b) $\int_{-\infty}^{+\infty} \Psi^* \Psi dx = 1 \Rightarrow \int_{-\infty}^{-a} 0 dx + \int_{-a}^{+a} N^2 dx + \int_{+a}^{\infty} 0 dx = 1$ ✓

∴ $\int_{-a}^{+a} N^2 dx = 1 \Rightarrow N^2 [x]_{-a}^{+a} = 1$ ✓

$N^2 \cdot 2a = 1 \quad \therefore N = \frac{1}{\sqrt{2a}}$ ✓

c) $\langle x \rangle = \int_{-\infty}^{+\infty} \Psi^* x \Psi dx$ ✓

$\langle x \rangle = \int_{-a}^{+a} N x N dx = N^2 \int_{-a}^{+a} x dx$ ✓

$$\langle x \rangle = N^2 \left[\frac{x^2}{2} \right]_{-a}^{+a} = N^2 \left(\frac{a^2}{2} - \frac{a^2}{2} \right)$$

$$\langle x \rangle = 0 \quad \checkmark$$

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$$\langle x^2 \rangle = \int_{-\infty}^{+\infty} \Psi^* x^2 \Psi dx \quad \checkmark$$

$$\langle x^2 \rangle = \int_{-a}^{+a} N x^2 N dx \quad \checkmark$$

$$\langle x^2 \rangle = N^2 \int_{-a}^{+a} x^2 dx = N^2 \left(\frac{a^3}{3} + \frac{a^3}{3} \right)$$

$$\langle x^2 \rangle = N^2 \cdot \frac{2a^3}{3} = \frac{1}{2a} \cdot \frac{2a^3}{3} = \frac{a^2}{3} \quad \checkmark$$

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$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{a^2}{3} - 0^2}$$

$$\Delta x = \frac{a}{\sqrt{3}} \quad \checkmark$$

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TOTAL

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