Quantum Mechanics A PHY-319 Problems 9

Issue: Thursday 15th March 2012 **Hand-in:** Thursday 22nd March 2012
Hand-In questions **1** and **2**
Please post your solutions in the box outside the 1st floor departmental offices

QUESTION 1.

In spherical coordinates, and for a radial potential, *V(r)*, the time-independent Schrödinger equation can be written as:

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2\psi}{\partial\phi^2}\right)\right]+V(r)\psi=E\psi$$

a) Use the technique of separation of variables to obtain the angular equation, namely:

$$-\frac{1}{Y(\theta,\phi)}\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y(\theta,\phi)}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 Y(\theta,\phi)}{\partial\phi^2}\right\}=l(l+1)$$

b) Use the technique of separation of variables again to obtain the azimuthal angle equation, namely:

$$-\frac{1}{\Phi(\phi)}\frac{d^2\Phi(\phi)}{d\phi^2}=m_l^2$$

c) Hence prove that m_l has to be an integer.

QUESTION 2.

a) Use the Bohr formula to obtain the first four energy levels of a Hydrogen atom.
b) Write down <u>all</u> the corresponding values of n, l and m_l for each level.
c) Ignoring spin, write down the corresponding degeneracy of each level.
d) What is the degeneracy including spin?

Exercise Class Questions.

a) The technique of separation of variables is used repeatedly in quantum mechanics. It is based on writing a function of several variables as the product of several functions, each of a single variable, for example:

$$F(p,q,r) = P(p)Q(q)R(r)$$

Use the technique of separation of variables to obtain the time-independent Schrödinger equation from the time-dependent Schrödinger equation.

b) Consider an infinite one-dimensional potential well of length L , centred at the origin. The potential can be expressed as:

$$V(x) = 0 \text{ for } -\frac{L}{2} \le x \le \frac{L}{2},$$

and $\quad V(x) = \infty$ elsewhere.

The positive and negative parity normalised wavefunctions for such a well are given by:

Positive parity $\Psi_n(x,t) = \sqrt{\frac{2}{L}}\cos\left(\frac{n\pi}{L}x\right)e^{\frac{-iE_nt}{\hbar}}$ for $-\frac{L}{2} \le x \le \frac{L}{2}$ with $n = 1,3,5...$

$= 0 \quad$ elsewhere

Negative parity $\Psi_n(x,t) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)e^{\frac{-iE_nt}{\hbar}}$ for $-\frac{L}{2} \le x \le \frac{L}{2}$ with $n = 2,4,6...$

$= 0 \quad$ elsewhere

i) Obtain the probability density functions for both positive and negative parity solutions.

ii) A state, $\Psi(x,t)$, is described by the wavefunction:

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\{\Psi_1(x,t) + \Psi_2(x,t)\}.$$

Prove that its probability density function is given by:

$$|\Psi|^2 = \frac{1}{L}\left\{\cos^2\frac{\pi x}{L} + \sin^2\frac{2\pi x}{L} + 2\cos\frac{\pi x}{L}\sin\frac{2\pi x}{L}\cos\left(\frac{(E_2 - E_1)t}{\hbar}\right)\right\}$$

How is this probability density function fundamentally different to that obtained in part (i)?