Quantum Mechanics A PHY-319 Problems 7

Issue: Tuesday 6th March 2012 **Hand-in:** 16:00 Thursday 15th March 2012
Hand-In questions **1** and **2**
Please post your solutions in the box outside the 1st floor departmental offices

QUESTION 1.
The tip of a scanning tunnelling microscope is oscillated about its average position with amplitude 2Å, at a given distance, *z*, and hence average tunnelling current, *I(z)*; this leads to an oscillation in the tunnelling current, amplitude ΔI ...
Given that:

$$|\Delta I| = 2\kappa \Delta z I(z) \qquad \text{(from lecture notes)}$$

where $\kappa = \sqrt{\dfrac{2m_e\Phi_{HOPG}}{\hbar^2}}$, and Φ_{HOPG} the work function of the substrate (Highly Ordered Pyrolytic Graphite)

a) Plot two suitable graphs, one using the data below (obtained previously by Dr Scott) and one using the data obtained in Nanovision on the 6th March. Hence obtain the average work function of HOPG.

Note: Δz = 0.2nm

I(L)/nA	ΔI/nA
0.1	0.4
0.2	0.9
0.3	1.5
0.4	1.7
0.5	2
0.6	2.4
0.7	3.1
0.8	3.4
0.9	4.3
1.0	4.8

b) Draw two energy-position diagrams for the tunnelling scenario demonstrated; one under no bias and one with the platinum tip electrically connected (shorted) to the substrate. What further information do you require to make the diagram accurate? If we wanted to measure the work function of the HOPG, how would we modify the second set-up?

QUESTION 2.
The tunnelling current versus tip-substrate distance can be measured directly by withdrawing the tip from the substrate rapidly (fast compared to the constant current feedback loop). Over a limited range of distances this will obey the theoretical form:

$I(z) = I_0 e^{-2\kappa L}$ where $\kappa = \sqrt{\dfrac{2m\varphi}{\hbar^2}}$, φ is the barrier height and *z* the tip-substrate distance. One such data set has been obtained using an STM with a platinum tip and an HOPG substrate, the data has been saved in the computing and tools section of the course website as QMA-STM-I-z.txt. Using this file, plot a suitable graph and calculate φ. How does this compare to the values obtained in question 1?

Exercise Class Questions.

For an infinite square well potential (lying between 0 and L) the energy eigenstates are:

$$\Psi_n(x,t) = N\sin\left(\frac{n\pi}{L}x\right)e^{\frac{-iE_nt}{\hbar}} \quad \text{for } 0 \le x \le L \quad \text{with } n = 1,2,3...$$
$$= 0 \qquad \text{elsewhere}$$

a) By substituting into the Time-Dependent Schrödinger Equation (TDSE) inside the well, obtain the energy eigenvalue, E_n, for which $\Psi_n(x,t)$ is a solution. (Why don't you need the value of the constant, N, for this calculation?)

b) Calculate the normalisation constant, N.

c) An ensemble measurement is made of the particle's position when in the state $\Psi_n(x,t)$. Calculate the average position, $\langle x \rangle$, obtained.

d) Calculate the uncertainty Δx. (This is the "spread" in values of the individual measurements of the position in the ensemble measurement of part (c)).

e) Calculate the momentum uncertainty, Δp, given that $\hat{P} = -i\hbar\frac{\partial}{\partial x}$.

Useful "stuff"

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$
$$\int x\cos(ax)dx = \frac{x\sin(ax)}{a} + \frac{\cos(ax)}{a^2}$$
$$\int x^2\cos(ax)dx = \frac{x^2\sin(ax)}{a} + \frac{2x\cos(ax)}{a^2} - \frac{2\sin(ax)}{a^3}$$