Quantum Mechanics A PHY-319 Problems 6

Issue: Thursday 16th February 2012 **Hand-in:** 16:00 Thursday 1st March 2012
Hand-In question **1**
Please post your solutions in the box outside the 1st floor departmental offices

You will, undoubtedly, have a number of mid-term tests immediately following reading week, so just one (long) exercise and two weeks to get it done in this time.

QUESTION 1.

A beam of particles of energy E is incident from the left and passes over a finite potential well of width L and depth $-V_0$ as shown in figure 1.

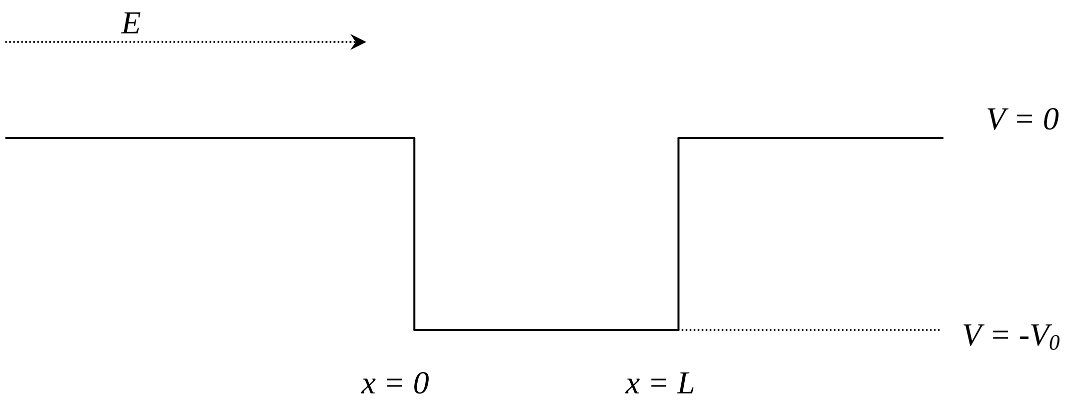

Figure 1: A beam of particles of energy *E* passes over a potential well

Prove that the transmission coefficient for such a situation is given by:

$$T = \frac{4q^2k^2}{\left(k^2 - q^2\right)^2 \sin^2(qL) + 4q^2k^2} \quad \text{where } q = \sqrt{\frac{2m}{\hbar^2}(V_0 + E)} \quad \text{and } k = \sqrt{\frac{2m}{\hbar^2}E}$$

NOTICE: There will be a 50 minute mid-term test at 13:00 on Tuesday the 28th of March in the usual lecture slot (that's the lecture in Engineering 3.25)

NB I have posted last year's mid-term test on the website at: http://ph.qmul.ac.uk/course/phy-319 so you can gain some practice.

Exercise Class Questions.

a) State Heisenberg's uncertainty relation in mathematical form.

b) What is the Born interpretation of the one-dimensional single particle normalised wave function $\Psi(x,t)$?

c) Write down the one-dimensional, time-dependent Schrödinger equation for a particle in a potential $V(x,t)$. Which condition allows us to derive the time-independent Schrödinger equation? Write down the time-independent Schrödinger equation.

d) **i)** For an observable, q, represented by the operator, $\hat{Q}$, write down an expression for its expectation value, $\langle q \rangle$.

ii) Write down an expression for the uncertainty in q, namely Δq.

e) A particle in a potential $V(x)$ is prepared in the eigenstate:

$$\Psi_n(x,0) = \psi_n(x)$$

What is the wavefunction of this particle at time t? i.e. what is $\Psi_n(x,t)$?

f) The momentum, p, is represented by the operator $\hat{P} = -i\hbar \frac{\partial}{\partial x}$. For a monochromatic matter wave given by $\Psi(x,t) = e^{\frac{i}{\hbar}(px-Et)}$, obtain the momentum eigenvalue.

g) Sketch the potential, the ground state and first excited state wave functions and their probability densities for a particle in:

i) an infinite square well.

ii) a finite square well.

iii) a simple harmonic oscillator

h) A system is prepared in the ground state of an infinite square well, width L; namely:

$$\Psi_1(x,t) = \sqrt{\frac{2}{L}} \cos\left(\frac{\pi x}{L}\right) e^{-\frac{iE_1 t}{\hbar}} \quad \text{for } -\frac{L}{2} \le x \le \frac{L}{2}$$

and $\Psi_1(x,t) = 0$ elsewhere…

i) Calculate the position uncertainty, Δx.

ii) Calculate the momentum uncertainty, Δp.

Useful "stuff"

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta), \quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\int x\cos(ax)dx = \frac{x\sin(ax)}{a} + \frac{\cos(ax)}{a^2}$$

$$\int x^2\cos(ax)dx = \frac{x^2\sin(ax)}{a} + \frac{2x\cos(ax)}{a^2} - \frac{2\sin(ax)}{a^3}$$