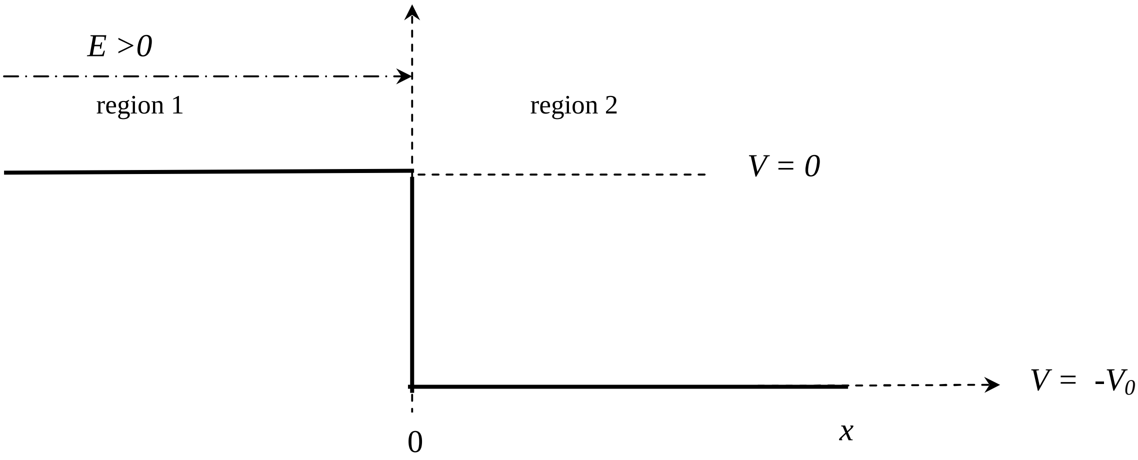
Quantum Mechanics A PHY-319 Problems 5

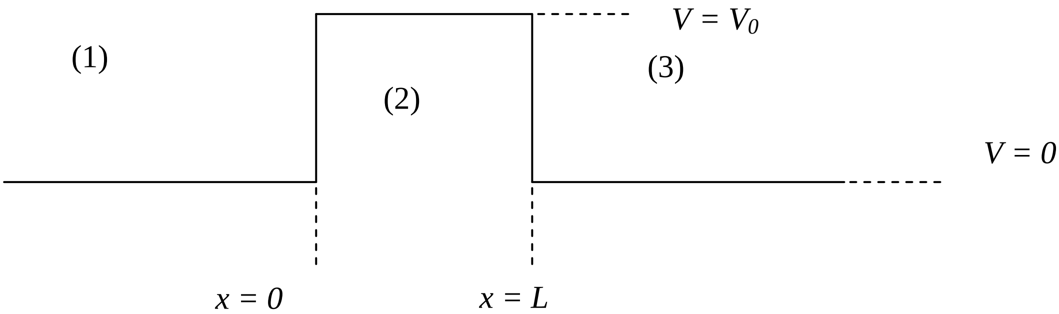
Issue: Thursday 9th February 2012 **Hand-in:** 16:00 Thursday 16th February 2012
Hand-In questions **1** and **2**
Please post your solutions in the box outside the 1st floor departmental offices

QUESTION 1.
A beam of particles of energy $E > 0$ is incident *from the left to the right* on the downward step potential shown below:



a) Write down the TISE in the two regions and obtain the general solutions.
b) Apply appropriate boundary conditions and obtain expressions for the transmission and reflection coefficients.
c) Given that $E = 32eV$ and $V_0 = 16eV$, calculate the transmission and reflection coefficients. Check they obey conservation of particles.

QUESTION 2.
A beam of particles of energy $E > 0$ is incident *from the left to the right* on the potential barrier shown below:



a) In the case where $E < V_0$, obtain solutions to the Schrödinger Equation in the three regions indicated. What boundary conditions would have to apply at $x = 0$ and $x = L$?
b) Repeat **Q2.a)** for the case $E > V_0$.

Exercise Class Questions.

A beam of free particles travelling from left to right can be modelled as a de Broglie matter wave:

$$\Psi(x,t) = Ae^{\frac{i}{\hbar}(px-Et)}$$

where A can be related to the number density of the incident particles, and the other symbols have their usual meanings.

1) Given that the particle flux can be expressed as:

$$j(x,t) = \frac{1}{m}\mathrm{Re}\left[\Psi^* \hat{P}\Psi\right]$$

obtain the flux for the de Broglie wave shown above.

2) Consider a beam of free particles of energy E travelling in one dimension, above a constant potential ($V = 0$).

a) Solve the Time Independent Schrödinger Equation and obtain a general solution for $\psi(x)$.

b) By considering the expression for $\Psi(x,t)$ when the potential is time-independent, show that the solution corresponds to two de Broglie matter waves, one travelling to the right and one travelling to the left.

3) By solving the Time Independent Schrödinger Equation for a beam of free particles of energy E travelling in one dimension, obtain the corresponding wavenumbers in the following two cases:

a) When the potential is constant, value V_0, where $E > V_0$.

b) When the potential is constant, value $-V_0$.